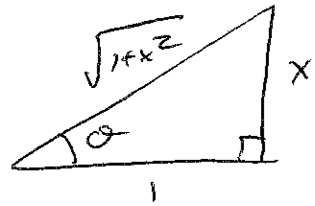
Name: Key

Math 221 Section 10336 *Exam 2* *June 4, 2012*

Follow the instructions for each question and show enough of your work so that I can follow your thought process. If I can't read your work or answer, you will receive little or no credit! There are 10 problems all of them each worth 5 points for a total of 50 points. There is also a bonus problem at the end worth 3 points.

1. Write $\sin(\tan^{-1} x)$ as an algebraic expression.

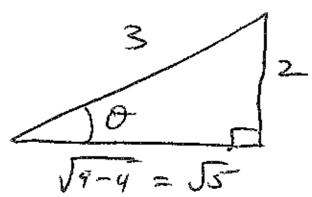
$\theta = \tan^{-1} x \qquad \tan\theta = x$



$$\sin(\tan^{-1} x) = \sin\theta = \frac{x}{\sqrt{1+x^2}}$$

2. Find the exact value of $\tan\left(\sin^{-1}\left(\frac{2}{3}\right)\right)$.

$\theta = \sin^{-1}\left(\frac{2}{3}\right) \qquad \sin\theta = \frac{2}{3}$



$$\tan\left(\sin^{-1}\left(\frac{2}{3}\right)\right) = \tan\theta = \frac{2}{\sqrt{5}}$$

For problems 3 - 8 differentiate each function.

3. $y = \tan^2(\sin\theta)$

$$y = (\tan(\sin\theta))^2$$

$$y' = 2\tan(\sin\theta)(\tan(\sin\theta))'$$

$$= 2\tan(\sin\theta)\sec^2(\sin\theta)(\sin\theta)'$$

$$= 2\tan(\sin\theta)\sec^2(\sin\theta)\cos\theta$$

4. $f(x) = x^2 \cos x$

$$f'(x) = 2x\cos x - x^2\sin x$$

5. $g(t) = \frac{e^t}{1+e^t}$

$$g'(t) = \frac{(e^t)'(1+e^t) - e^t(1+e^t)'}{(1+e^t)^2}$$

$$= \frac{e^t(1+e^t) - e^t e^t}{(1+e^t)^2} = \frac{e^t}{(1+e^t)^2}$$

6. $h(x) = x^5 \ln x$

$$h'(x) = 5x^4 \ln x + x^5\left(\frac{1}{x}\right)$$

$$= 5x^4 \ln x + x^4$$

7. $y = \arcsin(\sqrt{x})$

$$y = \arcsin(x^{\frac{1}{2}})$$

$$y' = \frac{1}{\sqrt{1-(x^{\frac{1}{2}})^2}} \left(x^{\frac{1}{2}}\right)'$$

$$= \frac{1}{\sqrt{1-x}} \cdot \frac{1}{2} x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}\sqrt{1-x}}$$

8. $g(t) = \tan^{-1}(t) + \tan^{-1}\left(\frac{1}{t}\right)$

$$g(t) = \tan^{-1} t + \tan^{-1}(t^{-1})$$

$$g'(t) = \frac{1}{1+t^2} + \frac{1}{1+\frac{1}{t^2}} (t^{-1})'$$

$$= \frac{1}{1+t^2} + \frac{1}{1+\frac{1}{t^2}} (-t^{-2})$$

$$= \frac{1}{1+t^2} - \frac{1}{\left(1+\frac{1}{t^2}\right)t^2} = \frac{1}{1+t^2} - \frac{1}{1+t^2} = 0$$

9. Compute the differential for $y = \sqrt{1+x^2}$.

$$y = (1+x^2)^{\frac{1}{2}}$$

$$y' = \frac{1}{2}(1+x^2)^{-\frac{1}{2}}(2x) = x(1+x^2)^{-\frac{1}{2}}$$

$$dy = \frac{x}{\sqrt{1+x^2}}\,dx$$

10. Find two numbers whose product is 81 and whose sum is a minimum.

$S = x + y \qquad xy = 81 \qquad$ so $\quad y = \frac{81}{x}$ d

$S(x) = x + \frac{81}{x}$, $S'(x) = 1 - \frac{81}{x^2} = \frac{x^2-81}{x^2} = \frac{(x-9)(x+9)}{x^2}$

$\underline{S'(x) = 0}$ $\qquad$ $\underline{S'(x) \text{ DNE}}$

$x = \pm 9$ $\qquad$ $x = 0$ $\quad$ $\cancel{x}$ thrown out

$S''(x) = \frac{162}{x^3}$, $S''(9) > 0 \Rightarrow x = 9$ min and $y = \frac{81}{9} = 9$

the #s are 9 and 9

11. (BONUS) We define the hyperbolic sine and hyperbolic cosine as follows:

$$\sinh x = \frac{e^x - e^{-x}}{2} \text{ and } \cosh x = \frac{e^x + e^{-x}}{2} .$$

Show that $\frac{d}{dx}(\sinh x) = \cosh x$ and $\frac{d}{dx}(\cosh x) = \sinh x$.

$$\frac{d}{dx}(\sinh x) = \frac{d}{dx}\left(\frac{e^x - e^{-x}}{2}\right) = \frac{e^x - e^{-x}(-1)}{2} = \frac{e^x + e^{-x}}{2} = \cosh x$$

$$\frac{d}{dx}(\cosh x) = \frac{d}{dx}\left(\frac{e^x + e^{-x}}{2}\right) = \frac{e^x + e^{-x}(-1)}{2} = \frac{e^x - e^{-x}}{2} = \sinh x$$