Name: Key

Math 221 Section 10336 — *Exam 1* — *Monday 21, 2012*

Follow the instructions for each question and show enough of your work so that I can follow your thought process. If I can't read your work or answer, you will receive little or no credit! There are 10 problems all of them each worth 5 points for a total of 50 points. There is also a bonus problem at the end worth 3 points.

For problems 1 and 2 find the limit if it exists.

1. $\lim_{x \to -4} \dfrac{x^2+2x-8}{x+4}$ $= \lim_{x \to -4} \dfrac{(x+4)(x-2)}{x+4} = \lim_{x \to -4} (x-2)$

$= -4-2 = -6$

2. $\lim_{x \to \infty} \dfrac{5x^3+3x^2-2}{4x^3-x^2+2x-1}$ $= \lim_{x \to \infty} \dfrac{5+\frac{3}{x}-\frac{2}{x^3}}{4-\frac{1}{x}+\frac{2}{x^2}-\frac{1}{x^3}}$

$= \dfrac{5}{4}$

3. Compute the derivative of $f(x) = \sqrt{x+3}$ using the definition of the derivative (4-step process). All other methods will have zero value.

$$f(x+\Delta x) = \sqrt{x+\Delta x+3}$$

$$f(x+\Delta x) - f(x) = \sqrt{x+\Delta x+3} - \sqrt{x+3}$$

$$= \left(\sqrt{x+\Delta x+3} - \sqrt{x+3}\right) \frac{\left(\sqrt{x+\Delta x+3} + \sqrt{x+3}\right)}{\left(\sqrt{x+\Delta x+3} + \sqrt{x+3}\right)}$$

$$= \frac{x+\Delta x+3-x-3}{\sqrt{x+\Delta x+3} + \sqrt{x+3}} = \frac{\Delta x}{\sqrt{x+\Delta x+3} + \sqrt{x+3}}$$

$$\frac{f(x+\Delta x) - f(x)}{\Delta x} = \frac{\Delta x}{\Delta x\left(\sqrt{x+\Delta x+3} + \sqrt{x+3}\right)} = \frac{1}{\sqrt{x+\Delta x+3} + \sqrt{x+3}}$$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{1}{\sqrt{x+\Delta x+3} + \sqrt{x+3}} = \frac{1}{\sqrt{x+3} + \sqrt{x+3}}$$

$$= \frac{1}{2\sqrt{x+3}}$$

For problems 4 - 7 differentiate each function.

4. $f(x) = x^6 + 3x^3 - \pi x^2 + 10x - e$

$f'(x) = 6x^5 + 9x^2 - 2\pi x + 10$

5. $y = 2x\sqrt{x^2+1}$

$y' = (2x)'\sqrt{x^2+1} + 2x\left[(x^2+1)^{\frac{1}{2}}\right]'$

$= 2\sqrt{x^2+1} + 2x\left(\frac{1}{2}(x^2+1)^{-\frac{1}{2}}(x^2+1)'\right)$

$= 2\sqrt{x^2+1} + x(x^2+1)^{-\frac{1}{2}}2x = 2\sqrt{x^2+1} + \frac{2x^2}{\sqrt{x^2+1}}$

6. $g(t) = \dfrac{3t-1}{5t^2+1}$

$$g'(t) = \frac{(3t-1)'(5t^2+1) - (3t-1)(5t^2+1)'}{(5t^2+1)^2}$$

$$= \frac{3(5t^2+1) - (3t-1)(10t)}{(5t^2+1)^2} = \frac{15t^2+3-30t^2+10t}{(5t^2+1)^2}$$

$$= \frac{-15t^2+10t+3}{(5t^2+1)^2}$$

7. $h(x) = \left(x + \dfrac{1}{x^2}\right)^{2012} = (x+x^{-2})^{2012}$

$$h'(x) = 2012(x+x^{-2})^{2011}(x+x^{-2})'$$

$$= 2012(x+x^{-2})^{2011}(1-2x^{-3})$$

$$= 2012\left(x+\frac{1}{x^2}\right)^{2011}\left(1-\frac{2}{x^3}\right)$$

8. Use implict differentiation to compute $\dfrac{dy}{dx}$ of $y^5 + x^2y^3 = 1 + x^4y$.

$$\frac{d}{dx}\left(y^5+x^2y^3\right)=\frac{d}{dx}\left(1+x^4y\right)$$

$$5y^4\frac{dy}{dx}+2xy^3+3x^2y^2\frac{dy}{dx}=4x^3y+x^4\frac{dy}{dx}$$

$$5y^4\frac{dy}{dx}+3x^2y^2\frac{dy}{dx}-x^4\frac{dy}{dx}=4x^3y-2xy^3$$

$$\frac{dy}{dx}\left(5y^4+3x^2y^2-x^4\right)=4x^3y-2xy^3$$

$$\frac{dy}{dx}=\frac{4x^3y-2xy^3}{5y^4+3x^2y^2-x^4}$$

9. Compute the equation of the tangent line to the curve $f(x) = x + \dfrac{1}{x}$ at $x = 1$.

$f(x)=x+x^{-1}$ $\quad f(1)=1+\frac{1}{1}=2$ $\quad$ pt $(1,2)$

$f'(x)=1-x^{-2}$ $\quad m=f'(1)=1-\frac{1}{1^2}=0$

$y-2=0(x-1) \Rightarrow y=2$

10. Do a complete curve sketching analysis of the curve $f(x) = x^3 + 6x^2 + 9x + 1$. In other words find all critical numbers, critical points, inflection points, intervals on which $f(x)$ is increasing/decreasing, intervals of concave up/concave down, and all relative maximum and minimum and finally sketch the curve.

$$f'(x) = 3x^2 + 12x + 9$$
$$= 3(x^2 + 4x + 3) = 3(x+3)(x+1)$$

$$f''(x) = 6x + 12$$
$$= 6(x+2)$$

$\underline{f'(x) = 0}$
$3(x+3)(x+1) = 0$
$x = -3, -1$

$\underline{f' \text{ DNE}}$
None

$\underline{f''(x) = 0}$
$6(x+2) = 0$
$x = -2$

$f(-3) = 1$, $f(-1) = -3$
$f(-2) = -1$

$\nearrow \quad \searrow \quad \nearrow$
$+ \quad - \quad +$
$-3 \quad -1$

$f'(x) = 3(x+3)(x+1)$

$- \quad +$
-2

$f''(x) = 6(x+2)$

$f'(-4) > 0 \quad f'(0) > 0$
$f'(-2) < 0$

$f''(-3) < 0 \quad f''(0) > 0$

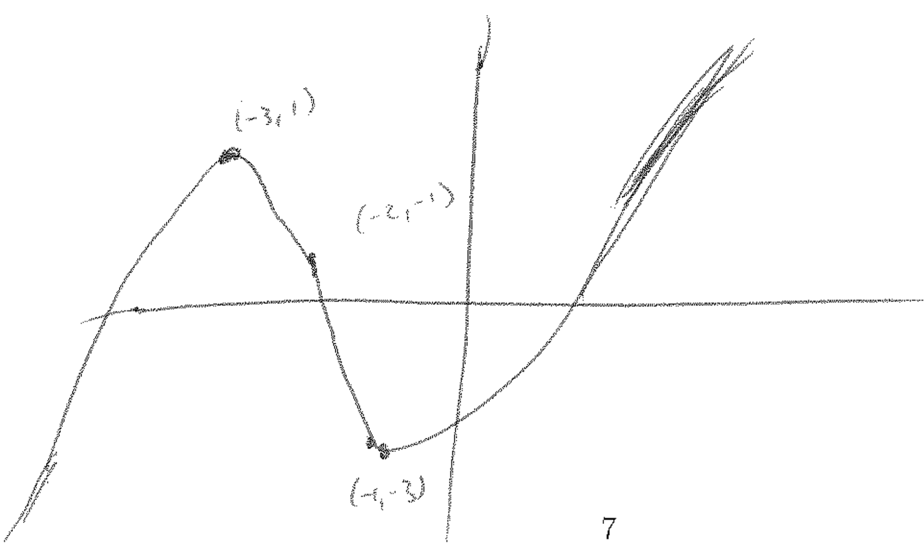
$\underline{\text{C. \#}}$
$x = -3, -1$

$\underline{\text{C. P.}}$
$(-3, 1), (-1, -3)$
local max local min

$\underline{\text{I. P.?}}$
$(-2, -1)$ ✓

$f \nearrow : (-\infty, -3) \cup (-1, \infty)$
$\searrow : (-3, -1)$

$f \cup : (-2, \infty)$
$\cap : (-\infty, -2)$



11. (BONUS) Compute $\frac{d^n y}{dx^n}$ for $y = x^n$.

$$\frac{d^n y}{dx^n} = n!$$