Math 166 Section 19061   *Practice Exam 3*   *November 10, 2011*

Follow the instructions for each question and show enough of your work so that I can follow your thought process. If I can't read your work or answer, you will receive little or no credit!

For problems 1 and 2, find the length of the curve.

1. $r = \sin^3\left(\dfrac{\theta}{3}\right)$ for $0 \leq \theta \leq \pi$.

2. $r = \dfrac{1}{\theta}$ for $\pi \leq \theta \leq 2\pi$.

For problems 3 - 5, determine if the following sequences are convergent or divergent, If they are convergent find their limits.

3. $\left\{ \dfrac{(n+2)!}{n!} \right\}_{n=1}^{\infty}$

4. $a_n = \dfrac{(-1)^n n^4}{n^5 + 3n^3 + 34}$

5. $\left\{ \dfrac{\ln(3n)}{\ln(5n)} \right\}_{n=1}^{\infty}$

For problems 6 - 9, find the sum of the following series.

6. $\displaystyle\sum_{n=1}^{\infty} \frac{3}{n(n+3)}$

7. $\displaystyle\sum_{n=1}^{\infty} \left(\cos\left(\frac{1}{n^2}\right) - \cos\left(\frac{1}{(n+1)^2}\right) \right)$

8. $\displaystyle\sum_{n=1}^{\infty} \frac{\pi^{n+1}}{6^n}$

9. $\displaystyle\sum_{n=1}^{\infty} \frac{(-4)^n}{10^{n+1}}$

For problems 10 - 17, determine if the following series converges or diverges. Be sure to state which method you are using. If the series converges and you used the ratio or root test state whether the convergence is absolutely or conditionally.

10. $\displaystyle\sum_{n=1}^{\infty} \frac{e^{1/n}}{n^2}$

11. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln n}}$

12. $\displaystyle\sum_{n=1}^{\infty} \frac{n^2 + 1}{n^4 + 1}$

13. $\displaystyle\sum_{n=2}^{\infty} \frac{2 + \sin n}{\sqrt{n} - 1}$

14. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n (n^2 + 1)}{n^5 + 1}$

15. $\displaystyle\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{\ln n}$

16. $\displaystyle\sum_{n=1}^{\infty} \frac{(5n)^n}{(n!)^n}$

17. $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{2^n n!}{5 \cdot 8 \cdot 11 \cdots (3n+2)}$

For problems 18 - 21, find the radius of convergence and the interval of convergence of the following series.

18. $\displaystyle\sum_{n=1}^{\infty} \frac{2^n(2x-3)^n}{5^n n!}$

19. $\displaystyle\sum_{n=1}^{\infty} \frac{10^n(x-5)^n}{n^3}$

20. $\displaystyle\sum_{n=1}^{\infty} \frac{2^n(x-2)^n}{(n+2)!}$

21. $\displaystyle\sum_{n=0}^{\infty} \frac{4^n (x-4)^{2n}}{\sqrt{n+7}}$

22. Let $\displaystyle\sum_{n=1}^{\infty} a_n$ be a series whose n^{th}-partial sum is $s_n = \dfrac{3n+2}{5n+4}$. Find the terms of the series, a_n, and the sum of the series, $\displaystyle\sum_{n=1}^{\infty} a_n$.

23. Let $\displaystyle\sum_{n=1}^{\infty} a_n$ be a series whose n^{th}-partial sum is $s_n = \dfrac{\pi^2 n + 1}{6n+1}$. Find the terms of the series, a_n, and the sum of the series, $\displaystyle\sum_{n=1}^{\infty} a_n$.