Name:_____

Math 166 Section 19061 *Practice Exam 1* *September 12, 2011*

Follow the instructions for each question and show enough of your work so that I can follow your thought process. If I can't read your work or answer, you will receive little or no credit!

1. Let $f(x) = x + x^2 + e^x$. Find $(f^{-1})'(1)$.

2. Let $f(x) = x^3 + 3\sin x + 2\cos x$. Find $(f^{-1})'(2)$.

3. Find the area of the region bounded by the curves $y = e^x$, $y = e^{3x}$, $x = 0$, and $x = 1$.

4. Find the area of the region bounded by the curves $y = 1/x$, $y = x$, $x = 1$, and $x = 3$.

For problems 5 - 10 find the derivative of the given function.

5. $f(t) = t^2 \ln t$

6. $h(u) = 10^{\sqrt{u}}$

7. $y = \ln|\sec(5x) + \tan(5x)|$

8. $y = 3^{x \ln x}$

9. $v(r) = r \tan^{-1} r$

10. $f(x) = \ln\left(\frac{1}{x}\right) + \frac{1}{\ln x}$

For problems 11 - 14, find the limit of the following functions if they exist.

11. $\lim_{x \to 1^+} \left(\frac{x}{x-1} - \frac{1}{\ln x}\right)$

12. $\lim_{x \to 0} \frac{\tan(\pi x)}{\ln(1+x)}$

13. $\lim_{x\to\infty} x^3 e^{-x}$

14. $\lim_{x\to\infty} \left(1+\frac{3}{x}\right)^{2x}$

For problems 15 - 20, evaluate the following integrals.

15. $\int e^{3x}\cos(7x)\,dx$

16. $\int t^3 e^{-t^2}\ dt$

17. $\int \sin^6 x \cos^{13} x\ dx$

18. $\int x \sec x \tan x\ dx$

19. $\displaystyle\int \frac{dx}{x^2\sqrt{9x^2-1}}$

20. $\displaystyle\int_{\frac{3\pi}{2}}^{2\pi} \frac{\sin t}{\sqrt{1+\cos^2 t}}\, dt$

21. Show that

$$\cos\left(\tan^{-1}\left(\sin\left(\cot^{-1}(x)\right)\right)\right) = \sqrt{\frac{x^2+1}{x^2+2}}\ .$$

22. Let a and b positive numbers and define

$$f(x) = \int_a^b t^x dt \ .$$

Show that

$$f(x) = \frac{b^{x+1} - a^{x+1}}{x+1} \quad \text{when} \quad x \neq -1$$

and $f(-1) = \ln b - \ln a$. Also show that $f(x)$ is continuous at $x = -1$.