Review Problems for Test 1

MATH 2443-006, Spring 04

1. Let  \( f(x, y) = \sqrt{36 - 4x^2 - 9y^2} \)
   
   a) Describe and sketch the domain of  \( f \).
   
   b) Since  \( P(-1/2, 1) \) is in the domain, there is a level curve for  \( f \) at  \( C \) which passes through  \( P \). Find the value  \( C \).

2. Find the limit or show that it does not exist
   
   a)  \( \lim_{(x,y) \to (0,0)} \frac{(x-y)^2}{x^2+y^2} \)
   
   b)  \( \lim_{(x,y) \to (0,0)} \frac{x^3+y^3}{x^2+y^2} \)

3. Use linear approximation to approximate a suitable function  \( f(x,y) \) and thereby estimate the following
   
   \( (0.99e^{0.02})^8 \)

4. Consider the surface  \( xyz = 30 \)
   
   a) Find the plane tangent to the surface at the point  \( (2, 3, 5) \)
   
   b) Give a parametric equation for the line normal to the surface at  \( (2, 3, 5) \)

5. Given  \( z = e^r \cos \theta, \ r = st, \ \theta = \sqrt{s^2+t^2} \), find  \( \frac{\partial z}{\partial s} \) and  \( \frac{\partial z}{\partial t} \).

6. Find the maximum rate of change of  \( f(x, y) = x^2y + y^2z \) at the point  \( P(1, 2, -1) \) and the direction in which it occurs.

7. Compute the derivative of  \( f(x, y, z) = e^x + yz \) at  \( P(1, 1, 1) \) in the direction of  \( \mathbf{v} = (1, -1, 1) \).

8. Find the local maximum and minimum values and saddle points of the function  \( f(x, y) = 4xy + 2x^2y - xy^2 \).
9. Find the absolute maximum and minimum of \( f(x, y) = y\sqrt{x} - y^2 - x + 6y \) on the rectangle \( 0 \leq x \leq 9, \ 0 \leq y \leq 5 \).

10. Find 3 positive numbers whose sum is 100 and whose product is a maximum.

11. Use Lagrange multipliers to find the maximum and minimum values of the function \( f(x, y) = x^2 + y^2 \) subject to the constraint \( x^4 + y^4 = 1 \).