Review for Midterm II

MATH 2433-003, Honors

1. Test for convergence
   a) \[ \sum_{n=1}^{\infty} (-1)^{n-1} \frac{\ln n}{n} \]
   b) \[ \sum_{n=0}^{\infty} \frac{\cos(n\pi/3)}{n!} \]
   c) \[ \sum_{n=1}^{\infty} \frac{n!}{n^n} \]
   d) \[ \sum_{n=1}^{\infty} \left( \frac{n}{n+1} \right)^n n^2 \]

2. Find the radius of convergence and the interval of convergence of the power series
   a) \[ \sum_{n=1}^{\infty} \frac{(3x-2)^n}{n^3} \]
   b) \[ \sum_{n=1}^{\infty} \frac{(x+1)^n}{n(n+1)} \]

3. Use a power series to approximate the definite integral to 2 decimal places
   \[ \int_{0}^{1} \frac{dx}{1+x^4} \]

4. a) Find Taylor series for \( f(x) = \cos x \) at \( a = \pi/4 \).
   b) Find the radius of convergence of the series in a).
   b) Show that the Taylor series represent \( f(x) = \cos x \).

5. Find the Maclaurin series for \( \ln(1+x) \) and use it to calculate \( \ln 1.1 \) correct to two decimal places.

6. Find an equation of a sphere that passes through the origin and whose center is \((1, 2, 3)\).

7. Let \( \mathbf{a} = (1, -1, 2) \), \( \mathbf{b} = (3, 0, -2) \). Find \( |\mathbf{a}| \), angle \( \theta \) between \( \mathbf{a} \) and \( \mathbf{b} \), \( 2\mathbf{a} - \mathbf{b} \), \( \mathbf{a} \cdot \mathbf{b} \), \( \mathbf{a} \times \mathbf{b} \), \( \text{comp}_b \mathbf{a} \).

8. Find a unit vector that is orthogonal to both \( \mathbf{i} + \mathbf{j} \) and \( \mathbf{i} + \mathbf{k} \).
9. Find a vector orthogonal to the plane through the points \( P(1, 0, -1) \), \( Q(2, 4, 5) \), \( R(3, 1, 7) \) and the area of the triangle \( PQR \).