Review Problems for Test I

Honors Calculus I

1) Let \( f(x) = \frac{1}{x}, \ g(x) = x^3 \) and \( h(x) = x^2 + 2 \). Find \( f \circ g \circ h \).

2) Express \( H(x) = \sin^4(\sqrt{x}) \) in the form \( f \circ g \circ h \).

3) Determine whether the following limits exist. If yes, evaluate.
   1. \( \lim_{t \to 2} \frac{t^2 + t - 6}{t^2 - 4} \);
   2. \( \lim_{h \to 0} \frac{(1+h)^4 - 1}{h} \);
   3. \( \lim_{x \to 2} \frac{|x-2|}{x-2} \);
   4. \( \lim_{x \to 0} \sqrt{x^2 + x^3} \sin \frac{2x}{x^4} \).

4) Prove each of the following statements using the \( \epsilon - \delta \) definition of a limit.
   1. \( \lim_{x \to 9} (4x + 11) = 19 \);
   2. \( \lim_{x \to 0} x^3 = 0 \).

5) Find the points at which \( f(x) \) is discontinuous
   \[ f(x) = \begin{cases} 
   2x - 2 & \text{if } x \leq -1 \\
   3x & \text{if } -1 < x < 1 \\
   2x + 1 & \text{if } x \geq 1 
   \end{cases} \]

6) Find the constant \( c \) that makes \( g(x) \) continuous on \(( -\infty, +\infty )\)
   \[ g(x) = \begin{cases} 
   x^2 - c^2 & \text{if } x < 4 \\
   cx + 20 & \text{if } x \geq 4 
   \end{cases} \]

7) Prove that the equation
   \[ x^5 - x^2 + 2x + 3 = 0 \]
   has at least one real root.

8) Find an equation of the tangent line to the curve at a given point.
1. \( y = 1/\sqrt{x} \), at \( P(4, \frac{1}{2}) \);

2. \( y = x^2 + 1 \), at \( P(1, 2) \).