Statistics (MATH 4733/5733) Spring 2012
Homework 5

Due: Wed. Feb. 29, start of class

Instructions: Please read the homework policies and guidelines posted on the course webpage. You may **not** use a calculator (or computer) **except** for basic arithmetic functions (e.g., $+, -, \times, \div, \log 2, e^{1/2}, \sqrt{10}$). Make sure to write your name and course number in the top right corner of your solution set, as well as the assignment number on top. Please staple your homework. Sections and exercises refer to the exercises in the required course text.

Exercises marked with an asterisk ($*$) are for students of 5733 only.

Reading

Read Section 5.5 and Notes on Cramér-Rao Inequality.

Conceptual questions

- What does sufficiency mean? How is this notion helpful in constructing efficient estimators?
- What does consistency mean?

Written Assignment

Section 5.6: 1, 2, 5, 8*, 10

Section 5.7: 2

Problem A. Suppose X is a random variable with pdf $f_X(x;\theta)$ where the unknown parameter θ is the mean. Let $X_1, \ldots, X_n$ be a random sample. Is the sample mean estimator $\hat{\theta} = \frac{1}{n}\sum X_i$ always unbiased? Give a proof or counterexample.

Problem B. Suppose X is a random variable with pdf $f_X(x;\theta) = \frac{\theta-1}{x^\theta}$ where $\theta > 1$. Is $\hat{\theta} = \frac{1}{n}\sum X_i$ sufficient? If not, find a sufficient statistic.

Problem C. Let X be uniform with pdf $f_X(x;\theta) = \frac{1}{\theta}$ for $0 \le x \le \theta$. Let $\hat{\theta}_n = \frac{n+1}{n} X_{\max}$ where $X_{\max} = \max\{X_1, \ldots, X_n\}$.

(i) Is $\hat{\theta}_n$ consistent?

(ii) How large should n be to say that there is a 95% chance an estimate θ_e using $\hat{\theta}_n$ is within 2% accuracy of θ? [*Hint:* Estimate how large n should be using Chebyshev's inequality and Example 5.4.7. In fact n doesn't need to be this large, but it's not as easy to calculate this exactly.]

Problem D. Do Exercise 5.7.3 (a) and (b) and

(c) Is the sample mean $\hat{\lambda}_n = \frac{1}{n}\sum_{i=1}^n Y_i$ consistent for $1/\lambda$?

[**Correction**: in (a) and (b) of the text, it should also say "consistent for $1/\lambda$" as opposed to "consistent for λ."]