Probability (MATH 4733 - 01) Fall 2011
Bonus Problem Set

Due: Fri. Dec. 16, noon, PHSC 924

Instructions: Don't forget your name. **You may use a computer except where stated.**

Bonus Problems

Each problem might be worth something, depending on how much you put into it. (There is no are hard point values, but a full set of immaculate solutions could bump you up 1 or 2 letter grades.)

Warning: While I think doing these problems will help increase your understanding of the material significantly, do not spend all your time on these problems at the expense of preparing for the final exam—that is more important for your final grade.

Problem A. Consider a lottery in which 6 out of 80 balls, numbered 1 through 80, are chosen one at a time with an air mix lottery machine. You decide to model a single ball choice by a continuous uniform distribution X, and the lottery results as a random sample of size 6. With this model, what would you expect the lottery results to be? Does this give you any advantage in the actual lottery?

Problem B. Suppose that during normal business hours, a certain company fields about 100 calls per hour. Each call, once answered by a customer support representative, lasts an average of 3 minutes. If all customer service representatives are busy with other callers when you call, you are so informed by a machine and are put on hold until a representative is available. Calls are answered in the order they are received.

How many customer service representatives need to be employed so that 90% of the callers wait less than 5 minutes before speaking to a human? (Assume there are no "peak times.") How long should an average caller expect to wait before speaking to a human?

Problem C. Suppose X has a normal distribution with mean $\mu = 0$ and standard deviation σ Without a calculator, computer or table (i.e., from first principles—namely, calculus), estimate $P(\sigma < X < 2\sigma)$ correct to 3 decimal places.

Problem D. Let X be a continuous random variable with uniform distribution on $[0, 1]$ and Y be a continuous random variable with exponential distribution $f_Y(y) = e^{-y}$, $y > 0$. Denote their means and standard variations by μ_X, μ_Y, σ_X and σ_Y.

Let $X_1, \ldots, X_n$ be a random sample of size n for X and $Y_1, \ldots, Y_n$ be a random sample of size n for Y. Consider the normalized averages

$$X^{(n)} = \frac{X_1 + \ldots + X_n - n\mu_X}{\sqrt{n}\sigma_X}$$

and

$$Y^{(n)} = \frac{Y_1 + \ldots + Y_n - n\mu_Y}{\sqrt{n}\sigma_Y}.$$

Let

$$X^* = \lim_{n\to\infty} X^{(n)} \quad \text{and} \quad Y^* = \lim_{n\to\infty} Y^{(n)}.$$

Which do you expect to converge faster, on the level of pdfs? Graph the pdfs for $X^{(n)}$, $Y^{(n)}$ for $1 \le n \le 10$, as well as X^* and Y^*. How does this confirm/change your answer?

To quantify the rate of convergence, one could look at the errors

$$\varepsilon_{X,n} = \int_{-\infty}^{\infty} |f_{X^{(n)}}(x) - f_{X^*}(x)| dx$$

and

$$\varepsilon_{Y,n} = \int_{-\infty}^{\infty} |f_{Y^{(n)}}(y) - f_{Y^*}(y)| dy.$$

Compute $\varepsilon_{X,n}$ and $\varepsilon_{Y,n}$ for $1 \leq n \leq 10$.