Number Theory Fall 2009
Homework 2
Due: Fri. Sep. 11, start of class

Instructions: Read the homework guidelines and policies. Feel free to use a calculator or computer for the computational problems.

Written assignment

1.7 The Diophantus Chord Method

Exercise 1.6. *Use the rational slope method to find all rational points on the ellipse* $X^2 + 2Y^2 = 1$.

1.8 Gaussian Integers

Exercise 1.7. *Exercises 1.8.4, 1.8.5, 1.8.6.*

2.1 The gcd by subtraction

Exercise 2.1. *Exercises 2.1.1, 2.1.3, 2.1.5.*

Exercise 2.2. *Compute* $\gcd(84, 63)$ *using the method in Section 2.1. Write out each step.*

2.2 The gcd by division with remainder

Exercise 2.3. *Compute* $\gcd(42, 8)$ *using the division method. Write out each step.*

2.3 Linear representation of the gcd

Exercise 2.4. *Exercise 2.3.2. (You may use either the tableau method, the method in the text, or any other equivalent method you like. Just write down each step and explain your method if it is not one of two mentioned above.)*

2.5 Consequences of unique prime factorization

Proposition 2.1. *If N is nonsquare, then $\sqrt{N}$ is irrational.*

Exercise 2.5. *Does (the proof of) Proposition 2.1 (cf. Section 2.5) require that the prime factorization is unique? Explain.*

Exercise 2.6. *Prove the following (slighly generalized) assertion from Section 2.5: Unique prime factorization implies that each prime power divisor of a natural number n (i.e., a prime power that divides n) actually appears in the prime factorization of n. (Hint: it's easy, but you need to write down what it means to be a divisor.)*

Exercise 2.7. *Exercises 2.5.1 and 2.5.2.*

2.6 Linear Diophantine equations

Exercise 2.8. *Exercises 2.6.1, 2.6.2, 2.6.3, 2.6.4.*