

Calculus II - Fall 2015

Homework 12 (due Fri Dec 11)

Part I: Differential equations

A (30 points) Exercises 1–3 (10 points each) from the differential equations notes on the course page.

Part II: Review problems

In addition to the problems on the previous HW, the following problems constitute some of the main things I expect you to be able to do after completing the course. The types of problems below, along with those from HW 11, will probably account for at least 75% of the final exam.

B (5 points) Explain, in your own words, the basic idea for how to compute the area of a region in the plane, and how this leads to integrals (1–2 paragraphs, illustrations welcome).

C (5 points) Explain, in your own words, what the Fundamental Theorem of Calculus says and what is “fundamental” about it? (1 paragraph, illustrations welcome). (I am not asking for a formal statement with equations, but the main idea.)

D (10 points) Using integrals, prove the formula for the circumference of a circle of radius r .

E (10 points) (i) Graph xe^{-x} (compute relevant limits). Compute the area under this curve from $x = 0$ to ∞ or show it diverges.

(ii) Do the same for $x \ln x$.

F (30 points) Let R be the region in \mathbb{R}^2 (the xy -plane) bounded by $y = x^2 - 4$ and $y = 3x$.

(i) Sketch R and find its area.

(ii) Sketch the solid obtained by rotating R about the line $x = 0$, and find its volume.

(iii) Sketch the solid obtained by rotating R about the line $y = -5$, and find its volume.

G (10 points) Compute the following indefinite integrals:

(i) $\int \frac{dx}{x^3+x}$.

(ii) $\int \frac{x^2-1}{(x-1)^5} dx$.