Calculus II - Fall 2015
Homework 11 (due Fri Dec 4)

Version 1.2

This homework is about the geometry of conic sections and surfaces. I consider this to be *the* fundamental topic for the course, and as such, you should expect similar problems on the final. These problems incorporate both the new material on arc length and surface area, as well as some review material.

1. (30 points) Consider the curve C given by $y = f(x) = x^2$ on $[0, 4]$.

   (a) Find upper and lower estimates on the area under C using Riemann sums with 4 subdivisions. Draw pictures to illustrate your estimates.

   (b) Evaluate the area under C exactly using a limit of Riemann sums.

   (c) Estimate the length of C using 4 approximating line segments. Draw a picture to illustrate your estimate. Does your estimate give an upper bound or a lower bound on the length (or neither)?

   (d) Compute the arc length of C.

2. (35 points) Let E be the ellipse given by $\frac{x^2}{9} + \frac{y^2}{16} = 1$, and let S be the ellipsoid surface obtained by rotating E about the x-axis.

   (a) Draw the ellipse and compute the area enclosed by E.

   (b) Write down an integral expressing the arc length (circumference) of E. (The integral will essentially be a so-called *elliptic integral*, for which there is no elementary expression in general.)

   (c) Compute the volume enclosed by S.

   (d) Compute the surface area of S.

3. (35 points) Let R be the region in the plane bounded by hyperbola $y^2 - x^2 = 1$ bounded between the lines $x = -1$ and $x = 1$, and let S be the hyperboloid obtained by rotating R about the x-axis.

   (a) Draw R and compute its area.

   (b) Write a formula for the length of its (4-sided) boundary in terms of integrals (again one gets elliptic integrals).

   (c) Compute the volume of S.

   (d) Compute the surface area of S—note this includes two "lids" on the sides.

Note that you can compute surface area in 2 and 3, even though you can't compute the associated arc length (in an elementary way).