Linear Algebra (MATH 3333) Spring 2011 Section 4
Midterm Practice Problems

Throughout this set, V denotes a vector space. All of the exercises are fundamental, though for preparation purposes, I've included many more than I will be able to fit on an exam. I've starred a few which I think you should pay special attention to.

1. Go over the problems on your quiz and previous homeworks, and make sure you can do them correctly.

True/False

Circle T or F.

2. T F Any two nonzero vectors in $\mathbb{R}^3$ are linearly independent.

3. T F A minimal spanning set for V is a basis for V.

4. T F Any subspace of $\mathbb{R}^2$ is either a line through the origin or $\mathbb{R}^2$.

5. T F The span of two nonzero vectors is either a line through the origin or a plane through the origin.

6. T F The set of polynomials in x of degree at most 5 form a vector space.

7. T F There is a linear transformation from $\mathbb{R}^2 \to \mathbb{R}^3$ whose image is the cone $x^2 + y^2 = z^2$.

8. T F There is a linear transformation from $\mathbb{R}^3 \to \mathbb{R}^2$ whose image is the line $y = x + 1$.

9. T F There is a linear transformation from $\mathbb{R}^2 \to \mathbb{R}^3$ whose image is the plane $z = x + y$.

10. T F There is a linear transformation from $\mathbb{R}^2 \to \mathbb{R}^3$ whose image is $\mathbb{R}^3$.

Questions

11. If $S = \{v_1, v_2, \ldots, v_k\} \subseteq V$, define span$(S)$.

12. With S as above, define what it means for S to be a basis of V.

13. With S as above, define what it means for S to be linearly independent.

14. Find two different bases for $\mathbb{R}^2$ (no proof needed).

15. Consider the basis $S = \{t^2 + 1, t + 1, 3t^2 - t\}$ for the space of polynomials of degree at most 2. If $[v]_S = \begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix}$, find v.

16. Consider the basis $S = \left\{ \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \end{pmatrix} \right\}$ of $\mathbb{R}^2$. If $v = \begin{pmatrix} 4 \\ 6 \end{pmatrix}$, find $[v]_S$.

Problems

Show your work.

17. Show that the set of vectors of the form $\begin{pmatrix} a \\ b \\ a + b \end{pmatrix}$ in $\mathbb{R}^3$ forms a subspace of $\mathbb{R}^3$. Find a basis for this space (no proof needed). What is it's dimension? Describe this space geometrically.

18. Do the same as the previous problem for the subset $\{(x, y, z) : x + y + z = 0\}$ of $\mathbb{R}^3$.

19. Is $\{(x, y, z) : 2x - 3y + z = 1\}$ a subspace of $\mathbb{R}^3$?

20.* Let $A = \begin{pmatrix} 1 & 1 & -1 \\ 1 & -1 & 1 \\ -1 & 1 & 1 \end{pmatrix}$.

(a) Find a basis for the image of A.
(b) Find a basis for the kernel of A.
(c) Determine rank A and nullity A.

21. Do the same as the previous problem for $A = \begin{pmatrix} 1 & 1 & 2 & -1 \\ 2 & -2 & 1 & 1 \\ -2 & -2 & -1 & 1 \\ 3 & 3 & 0 & -1 \end{pmatrix}$.

22. Find a linear transformation from $\mathbb{R}^2$ to $\mathbb{R}^3$ whose image is the set of vectors of the form $\begin{pmatrix} a \\ 2b - a \\ b \end{pmatrix}$.

23. Find a linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^3$ whose image is the plane $z = x + y$.

24.* Let $u, v \in V$. Show $\{u, v\}$ is linearly dependent if and only if $u = cv$ or $v = cu$ for some $c \in \mathbb{R}$.

25. Let $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$. Suppose A is a projection (i.e., its image is a line or a point). Show $\det A := ad - bc = 0$.

26.* Let $A : \mathbb{R}^4 \to \mathbb{R}^3$ be a linear transformation. Suppose the kernel of A is a plane in $\mathbb{R}^4$. What can you say about the image of A?