Linear Algebra (MATH 3333 - 04) Spring 2011
Homework 1

Due: Fri. Jan. 28, start of class

Instructions: Please read the homework policies and guidelines posted on the course webpage. You may **not** use a calculator (or computer). Make sure to write your name, course and section numbers in the top right corner of your solution set, as well as the assignment number on top. Please staple your homework. Sections and exercises refer to the exercises in the required course text.

Reading

Read Sections 1.2–1.4. Section 1.2 introduces some other applications of matrices not yet mentioned in class.

Conceptual questions

⋆ Why might one be interested in more general number systems than $\mathbb{R}$?

Written Assignment

Total: 100 points. Each problem is worth 10 points unless otherwise noted.

Section 1.2: 6, 8

Section 1.3: 13, 19

Section 1.4: (5 pts each) 2, 6, 8(a) 10 , 11, 12

Problem A. Recall Hamilton's quaternions are $\mathbb{H} = \{a + bi + cj + dk : a, b, c, d \in \mathbb{R}\}$, where i, j, k satisfy

$$i^2 = j^2 = k^2 = ijk = -1.$$

From the above relations, deduce the following

$$\begin{aligned} ij &= -ji = k \\ jk &= -kj = i \\ ki &= -ik = j. \end{aligned}$$

(Continued on next page)

Problem B. We may also model the quaternions as 4×4 real matrices by writing

$$a + bi + cj + dk = \begin{pmatrix} a & b & c & d \\ -b & a & -d & c \\ -c & d & a & -b \\ -d & -c & b & a \end{pmatrix}.$$

Consider the quaternions $\alpha, \beta \in \mathbb{H}$ given by $\alpha = 1 - i + 2j - k$ and $\beta = 2 + 2i - 3j + k$.

(i) Write $\alpha + \beta$ and $\alpha \cdot \beta$ in the form $a + bi + cj + dk$. (Use the relations from Problem A for $\alpha\beta$.)

(ii) Write α and β as 4×4 matrices, and compute $\alpha + \beta$ and $\alpha\beta$ using matrix addition and multiplication.

(iii) Check that both (i) and (ii) give you the answers for $\alpha + \beta$ and $\alpha\beta$.

Remark. One can also model the quaternions using 2×2 complex matrices, namely

$$a + bi + cj + dk = \begin{pmatrix} a + bi & c + di \\ -c + di & a - bi \end{pmatrix}.$$

If you prefer, you may do Problem B(ii) using this model, instead of the model in 4×4 real matrices.

Problem C. Prove Theorem 1.2(a) and (b) when A, B and C are all 2×2 matrices. Note: this mean you need to check these statements when A, B and C are arbitrary 2×2 matrices, not just check it in a specific example. To get started, write

$$A = \begin{pmatrix} a_1 & a_2 \\ a_3 & a_4 \end{pmatrix}, \quad B = \begin{pmatrix} b_1 & b_2 \\ b_3 & b_4 \end{pmatrix}, \quad C = \begin{pmatrix} c_1 & c_2 \\ c_3 & c_4 \end{pmatrix}.$$

(Alternatively, you can follow the approach on p. 35.)