Calculus III Honors Spring 2010
Homework 2
Due: Wed. Feb. 10, start of class

Instructions: Please read the homework policies and guidelines posted on the course webpage. You may *not* use a calculator (or computer) except where stated. Make sure to write your name, course and section numbers in the top right corner of your solution set, as well as the assignment number on top. Page/section numbers refer to the course text.

Written Assignment

Total: 100 points. Each problem is worth 10 points unless otherwise noted.

Section 11.2: 59, 61

Section 11.3: 8, 33, 51, 59

Section 11.4 5, 6, 13, 46

Bonus. Recall 2 sets A and B have the same cardinality if we can construct a one-to-one correspondence between the elements of A and the elements of B. Let $\mathbb{N} = \{1, 2, 3, \ldots\}$ be the set of natural numbers, $\mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$ be the set of integers and $\mathbb{Q} = \left\{\frac{a}{b} : a, b \in \mathbb{Z}, b \neq 0\right\}$ be the set of rational numbers.

(a) Show there is a one-to-one correspondence between $\mathbb{N}$ and $\mathbb{Z}$.

(b) Show there is a one-to-one correspondence between $\mathbb{Z}$ and $\mathbb{Q}$.

This shows $\mathbb{N}$, $\mathbb{Z}$ and $\mathbb{Q}$ all have the "same amount" of infinity. On the other hand, Cantor came up with a clever proof that $\mathbb{N}$ and $\mathbb{R}$ do not have the same cardinality. The existence of space-filling curves shows that $\mathbb{R}$ and $\mathbb{R}^2$ have the same cardinality. See the Wikipedia entry on space-filling curves for some pictures of how they can be constructed.