Linear Algebra (MATH 3333) Spring 2009 Section 2 Homework 9 Due: Wed. Apr. 15, start of class

Instructions: Please read the homework policies and guidelines posted on the course webpage. You may *not* use a calculator (or computer). Make sure to write your name, course and section numbers in the top right corner of your solution set, as well as the assignment number on top. Page/section numbers refer to the course text.

Reading

(Optional) Read Sections 6.3 and 6.5 for the book's more detailed treatment of what we are doing in lecture.

Written Assignment

Total: 100 points

Each problem is worth 20 points.

Section 4.8 (p. 267): 10

Section 6.5 (p. 413): 6

Problem A. Let $A = \begin{pmatrix} 1 & 1 \\ -2 & 4 \end{pmatrix}$. Let S be the standard basis for \mathbb{R}^2 and $T = \left\{ \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \end{pmatrix} \right\}$. Find (i) $P_{S \leftarrow T}$ (ii) $P_{T \leftarrow S}$ (iii) $[A]_S$

(iv) $[A]_T$.

Problem B. Repeat Problem A with

$$A = \begin{pmatrix} 1 & 0 & 3 \\ 0 & 1 & 0 \\ 3 & 0 & 1 \end{pmatrix},$$

S the standard basis for \mathbb{R}^3 , and

$$T = \left\{ \begin{pmatrix} 1\\0\\1 \end{pmatrix} \begin{pmatrix} 0\\1\\0 \end{pmatrix} \begin{pmatrix} 1\\0\\-1 \end{pmatrix} \right\}.$$

Problem C. Let A and B be two 2×2 matrices. Prove $\det(AB) = \det(A) \det(B)$ in two different ways:

(i) Use the formula for a 2×2 determinant.

(ii) Use the geometric interpretation of the determinant (cf. notes or Example 6 and Remark on p. 163).