Linear Algebra (MATH 3333) Spring 2009 Section 2
Homework 6

Due: Wed. Mar. 4th, start of class

Instructions: Please read the homework policies and guidelines posted on the course webpage. You may **not** use a calculator (or computer). Make sure to write your name, course and section numbers in the top right corner of your solution set, as well as the assignment number on top. Page/section numbers refer to the course text.

Reading

Sections 4.3 and 4.4.

Conceptual Questions (not to be turned in)

1. What is span$\{v_1, v_2\}$? Write down the elements and give a geometric description.

Written Assignment

Total: 100 points

Each problem is worth 10 points.

Notation: M_{mn} is the vector space of $m \times n$ matrices and P_k is the vector space of polynomials of degree $\leq k$.

Section 4.3 (pp. 206–207): 13, 27, 29, 30 (no justification needed for 30)

Section 4.4 (pp. 215–216): 1, 2, 5, 6 (no justification needed for any of these)

Section 4.5 (p. 226): 1, 2 (show your work)

Bonus 1. Prove Theorem 4.4 when S is an infinite set, i.e., if V is a vector space and S is a infinite subset, then

$$\text{span}(S) := \{a_1v_1 + a_2v_2 + \cdots a_kv_k | a_1, \ldots a_k \text{ in } \mathbb{R}, v_1, \ldots, v_k \text{ in } S\}$$

(the set of all *finite* linear combinations of elements of S) is a subspace W of V. Give an example of a vector space V and a subspace W where S needs to be infinite. (Hint: for the example, you will need to look at *infinite-dimensional* vector spaces—the ones we've seen so far are the space of all polynomials with real coefficients and the space of all continuous functions from $\mathbb{R}$ to $\mathbb{R}$.)

Bonus 2. Let V be any vector space and W be a subspace of V. Prove that $W = \text{span}(S)$ for some subset S (possibly infinite) of V. (Hint: Take $S = W$, so all you need to show is that span$(W) = W$. Recall that the usual procedure to show two sets A and B are equal is to show it in two steps: first $A \subseteq B$, then $B \subseteq A$. For one of these steps you can use Bonus 1.)