Calculus II Honors Fall 2009
Homework 1
Due: Wed. Sep. 2, start of class

Instructions: Please read the homework policies and guidelines posted on the course webpage. You may *not* use a calculator (or computer) except where stated. Make sure to write your name, course and section numbers in the top right corner of your solution set, as well as the assignment number on top. Page/section numbers refer to the course text.

Reading

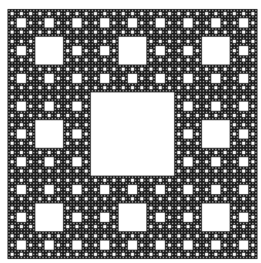
Sections 5.1 and 5.2 of the text.

Written Assignment

Total: 100 points. Each problem is worth 20 points.

Problem A: Let C be the curve in $\mathbb{R}^2$ given by $y = x^2$, $x = 0$ to $x = 1$. Approximate the length of C using 4 line segments. (Calculator allowed.)

Problem B: Let S_0 be the unit square. Divide S_0 into 9 subsquares, remove the middle one and call this S_1. Now S_1 is made up of 8 subsquares (each of side length 1/3). Divide each subsquare into 9 smaller subsubsquares, remove the middle subsubsquare from each of the 8 subsquares of S_1. Call this S_2. Repeat this process *ad infinitum* to get the *Sierpinski carpet* S (pictured below).



What do you think its area should be? Compute the inner area and outer area by approximating with smaller and smaller squares, and see if they agree or disagree. For the outer area you may want to (though don't have to) use the geometric series forumla: for $0 \leq r < 1$,

$$\sum_{n=0}^{\infty} r^n = \frac{1}{1-r}.$$

Problem C: (i) Prove

$$1^2 + 2^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

by induction.

(ii) Compute the upper and lower Riemann sums U_n and L_n for the function $f(x) = x^2$ on the interval $[a,b] = [0,1]$. Show that the limits are equal and determine $\int_0^1 x^2 dx$.

Problem D: Find an example of a function $f : [0,1] \to [0,1]$ such that the upper and lower Riemann sums U_n and L_n converge but $\lim U_n \neq \lim L_n$. Determine U_n and L_n. Nevertheless, can you guess what the "area under the curve" should be? Do you think there are any functions such that the upper and lower Riemann sums do not even converge as $n \to \infty$?

Problem E: Section 5.2: 33(d), 35, 50, 51.