1. Prove that every open set in \( \mathbb{R}^n \) is a countable union of open balls \( N_{\varepsilon_j}(x_j) \).

2. Prove that the Bolzano-Weierstrass property does not hold for \( C[a,b] \), the continuous functions on the interval \([a,b]\) with the supremum metric.

3. (Extreme Value Theorem) Prove that if \( E \) is compact and nonempty in metric space \((X,d)\) and if \( f : E \rightarrow \mathbb{R} \) is continuous, then the values
   \[
   M = \sup_{x \in E} f(x)
   \]
   and
   \[
   m = \inf_{x \in E} f(x)
   \]
   are both finite real numbers. Further, prove that there exist \( x_M \) and \( x_m \) in \( E \) such that \( f(x_M) = M \) and \( f(x_m) = m \).

4. Let \( E \) be a compact subset of metric space \((X,d)\) and let \( f, g : E \rightarrow \mathbb{R} \) be uniformly continuous on \( E \). Prove that the functions \( f + g \) and \( fg \) are uniformly continuous on \( E \). Where did the compactness of \( E \) come in?