Before you start these problems, make a list of the convergence tests and other properties of infinite series.

1. Prove the three parts of the Limit (or Asymptotic) Comparison Test. In each part, let \( a_j \geq 0 \) and \( b_j > 0 \) for all \( j \).

   a) If \( \lim_{j \to \infty} \frac{a_j}{b_j} = L \), where \( 0 < L < \infty \), then \( \sum_j a_j \) converges if and only if \( \sum_j b_j \) converges. (Hint: Use the Comparison Test.)

   b) If \( \lim_{j \to \infty} \frac{a_j}{b_j} = 0 \) and \( \sum_j b_j \) converges, then \( \sum_j a_j \) also converges. (Hint: Start with the Zero Test.)

   c) If \( \lim_{j \to \infty} \frac{a_j}{b_j} = \infty \) and \( \sum_j b_j \) diverges, then \( \sum_j a_j \) also diverges.

2. Determine whether or not each of the following series converges or diverges. Explain which test(s) you use in each.

   a) \( \sum_{j=1}^{\infty} \frac{j^{1/7}}{j} \)

   b) \( \sum_{j=1}^{\infty} \frac{3j^2 - \sqrt{j}}{j^4 - j^2 + 1} \)

   c) \( \sum_{j=1}^{\infty} \left( \frac{3 + (-1)^j}{3} \right)^j \)

3. Determine whether or not each of the following series converges absolutely, converges conditionally, or diverges. Explain which test(s) you use in each.

   a) \( \sum_{j=1}^{\infty} \frac{(-1)^j j^2}{2^j} \)

   b) \( \sum_{j=1}^{\infty} \frac{(-1)^j \sqrt{j}}{j + 1} \)