1. Let $f$ be continuous on $[a, b]$ and differentiable on $(a, b)$. Let $f(a) = f(b) = 0$. Prove that if $f(c) > 0$ for some $c$ in $(a, b)$, then there exist points $x_1$ and $x_2$ in $(a, b)$ such that $f'(x_1) > 0 > f'(x_2)$.

2. Let $f$ be twice differentiable on $(a, b)$ and let there be points $x_1 < x_2 < x_3$ in $(a, b)$ such that $f(x_1) > f(x_2)$ and $f(x_3) > f(x_2)$. Prove that there is a point $c$ in $(a, b)$ such that $f''(c) > 0$.

3. Suppose that $f$ is continuous and increasing on $[a, b]$. Prove that $\sup f(E) = f(\sup E)$ for every nonempty set $E \subseteq [a, b]$.

4. Evaluate $\lim_{x \to 0^+} x \ln x$.

5. Use the Inverse Function Theorem to prove that $\frac{d}{dx}(\arctan x) = \frac{1}{1 + x^2}$ for all $x \in (-\infty, \infty)$. 