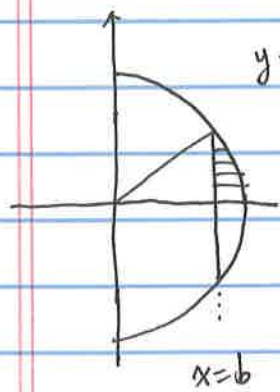
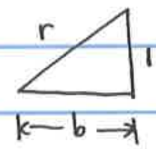
Calc II 2/22 (1)





$b^2 + 1 = r^2$

$$\int_0^1 \pi\left((\text{outer})^2 - (\text{inner})^2\right) dy$$

$$= \int_0^1 \pi\left(\left(\sqrt{r^2-y^2}\right)^2 - b^2\right) dy$$

$$= \int_0^1 \pi\left(r^2 - y^2 - b^2\right) dy$$

$$= \int_0^1 \pi\left(r^2 - b^2 - y^2\right) dy$$

$$= \int_0^1 \pi\left(1 - y^2\right) dy$$

$$= \pi \cdot \left(y - \frac{y^3}{3}\right)\Big|_0^1$$

$$= \pi\left(1 - \frac{1}{3}\right) = \frac{2\pi}{3}$$

__Fact__ An object of 10 kg has weight

$$10 \text{ kg} \cdot 9.8 \text{ N/kg} = 98 \text{ N}$$

__Fact__ Work = Force · Distance travelled

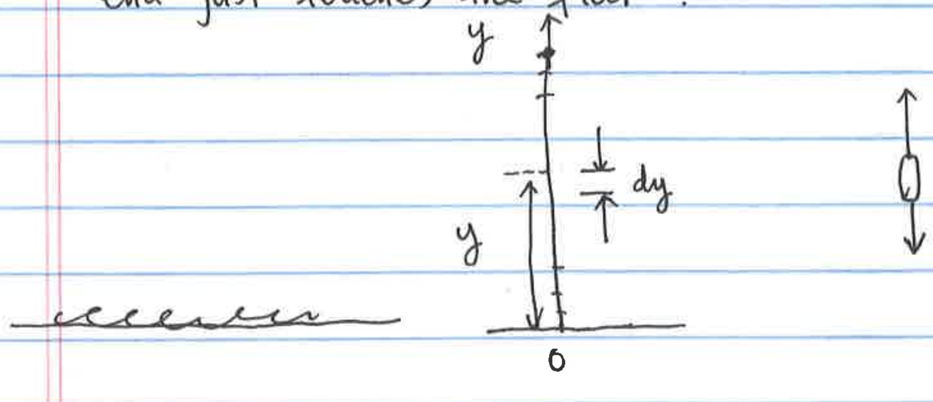
__Ex1__ How much work does it take to lift an object of 10 kg up by 1 m?

$$98 \text{ N} \cdot 1 \text{ m} = 98 \text{ N}\cdot\text{m} = 98 \text{ J}$$

(Joules)

2/22 ②

Ex2 A chain of weight 10 kg and length 2 m lies on the ground. How much work does it take to lift it so that the lower end just touches the floor?



length	weight
2 (m)	98 (N)
dy	

Guess #1) $\frac{2}{dy} \cdot 98$

Guess #2) $\frac{dy}{2} \cdot$

$\int$ work done to each dy-piece

$= \int (\text{weight of the dy-piece}) \cdot y$

$= \int_0^2 \frac{98 \cdot dy}{2} \cdot y$

$= \frac{49}{2} \cdot \frac{y^2}{2} \Big|_0^2$

$= 49 \cdot \frac{2^2}{2 \cdot 2} = 49 \text{ J}$

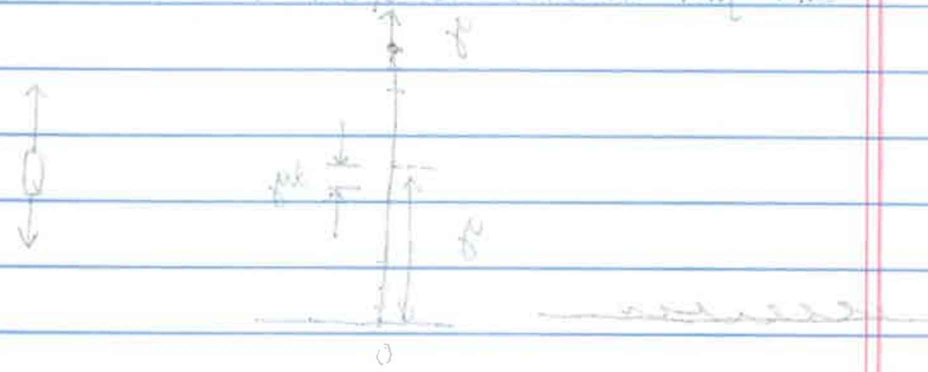
2/22 (2)

Ex 2. A chain of weight 10 kg and length 2m lies on the ground. How much work does it take to lift it so that the lower end just touches the floor?

length	weight
2 (m)	98 (N)
dy	:

(Guess #1) $\frac{2}{dy}\cdot 98$

(Guess #2) $\frac{dy}{2}$ ...



$\int$ work done to each dy-piece

$= \int (\text{weight of the } dy\text{-piece}) \cdot y$

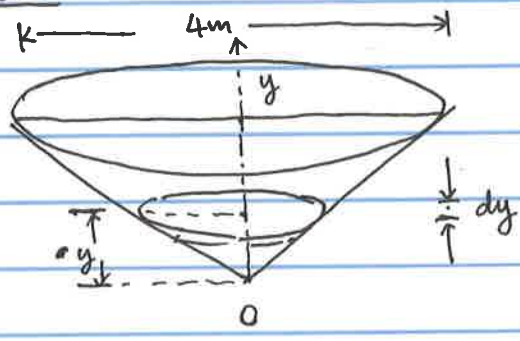
$= \int_0^2 \frac{98 \cdot dy}{2} \cdot y$

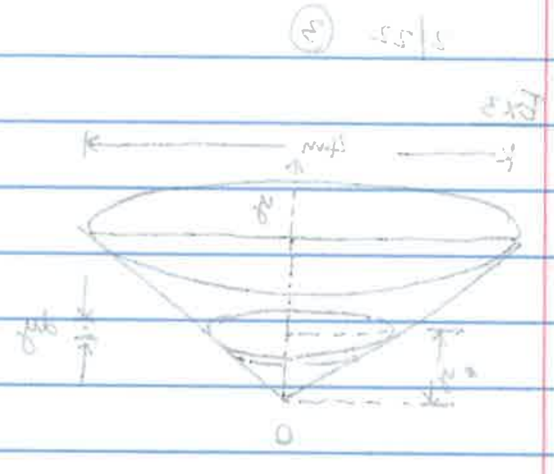
$= \frac{49}{2} \cdot \frac{y^2}{2} \Big|_0^2$

$= 49 \cdot \frac{2^2}{2 \cdot 2} = 49 \text{ J}$

2/22 (3)

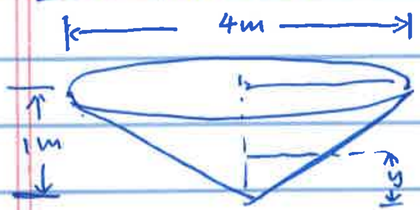
Ex3





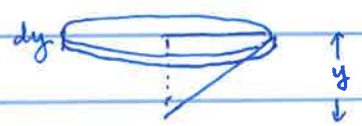
Calc II 2/26 ①

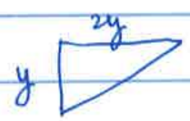
Ex 1 Find the work to fill a ~~tank~~ tank with water using the density of water $10^3\ kg/m^3$, $(1\ g/cm^3)$



$$\int \text{gravity of the disc} \cdot \underbrace{y}_{\text{distance traveled}}\ dy$$

$$= \int \underbrace{\underbrace{\underbrace{\pi r^2 \cdot dy}_{\text{Volume}} \cdot 10^3}_{\text{mass}} \cdot 9.8}_{\text{gravity}} \cdot y \ dy$$





$$= \int \pi (2y)^2 dy \cdot 10^3 \cdot 9.8\, y$$

slope of hypotenuse $= \frac{1}{2}$

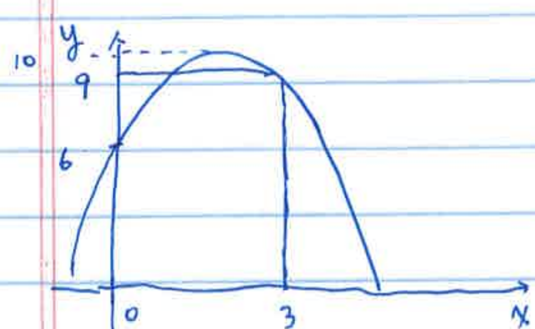
$r = 2y$

$$= \pi \int_0^1 4y^3 dy \cdot 9.8 \cdot 10^3$$

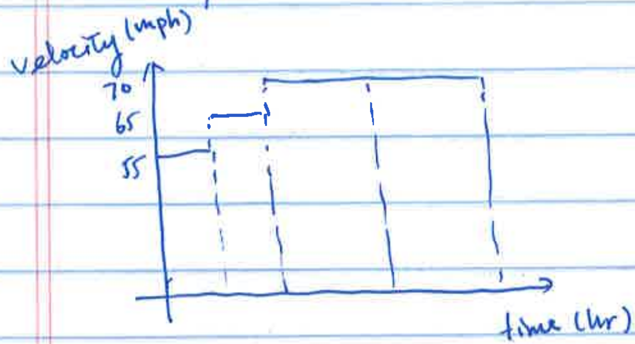
$$= \pi \cdot \frac{4y^4}{4} \cdot 9.8 \cdot 10^3 \Big|_0^1$$

$$= 9800\pi \ (J)$$

Ex 2 Find the average of the function $y = -x^2 + 4x + 6$ over $[0, 3]$



2/26 ②



distance = $55 \cdot 0.5 + 65 \cdot 0.5 + 70 \cdot 2$

distance = area under the graph of velocity

$$\text{average} = \frac{\text{distance}}{\text{time}} = \frac{\int \text{velocity}}{\text{time}}$$

Ex2.

$$\text{average} = \frac{\int_0^3 (-x^2+4x+6)\,dx}{3}$$

$$= \frac{-\frac{x^3}{3} + \frac{4x^2}{2} + 6x \Big|_0^3}{3}$$

$$= \frac{-9+18+18}{3} = \frac{27}{3} = 9$$

Find the intersection of the "line at the average height" & the graph

$$-x^2+4x+6 = 9$$

$x=1$ and $x=3$.

You could always find such intersections.