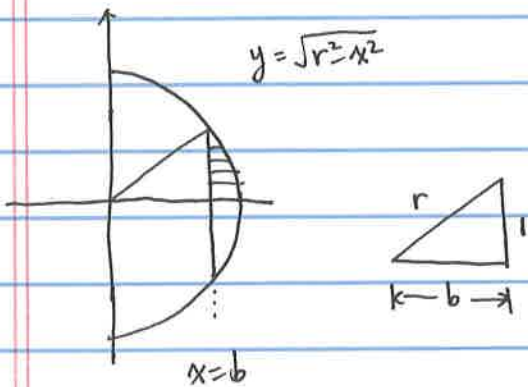


Calc II 2/22 ①



$$b^2 + 1 = r^2$$

$$\begin{aligned} & \int_0^1 \pi \left((\text{outer})^2 - (\text{inner})^2 \right) dy \\ &= \int_0^1 \pi \left((\sqrt{r^2 - y^2})^2 - b^2 \right) dy \\ &= \int_0^1 \pi (r^2 - y^2 - b^2) dy \\ &= \int_0^1 \pi (r^2 - b^2 - y^2) dy \\ &= \int_0^1 \pi (1 - y^2) dy \\ &= \pi \left(y - \frac{y^3}{3} \right) \Big|_0^1 \\ &= \pi \left(1 - \frac{1}{3} \right) = \frac{2\pi}{3} \end{aligned}$$

Fact An object of 10 kg has weight
 $10 \text{ kg} \cdot 9.8 \text{ N/kg} = 98 \text{ N}$

Fact Work = Force \cdot Distance travelled

Ex1 How much work does it take to lift an object of 10 kg up by 1 m?

$$98 \text{ N} \cdot 1 \text{ m} = 98 \text{ N} \cdot \text{m} = 98 \text{ J}$$

(Joules)

Case II

$$\int_0^1 \pi \left(\left(\frac{r}{d} \right)^2 - \left(\frac{r}{d} \right)^2 \right) dx$$

$$\int_0^1 \pi \left(\left(\frac{r}{d} \right)^2 - \left(\frac{r}{d} \right)^2 \right) dx =$$

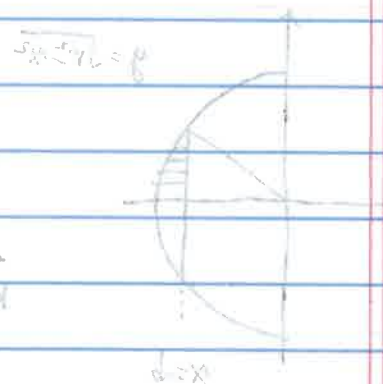
$$\int_0^1 \pi \left(\frac{r^2}{d^2} - \frac{r^2}{d^2} \right) dx =$$

$$\int_0^1 \pi \left(\frac{r^2}{d^2} - \frac{r^2}{d^2} \right) dx =$$

$$\int_0^1 \pi (1 - 1) dx =$$

$$\int_0^1 \pi \left(\frac{r^2}{d^2} - \frac{r^2}{d^2} \right) dx =$$

$$\frac{\pi r^2}{d^2} = \left(\frac{1}{3} - 1 \right) \pi =$$



$$r^2 = 1 - x^2$$

Force = weight of 10 kg = 98 N

Work = Force × Distance travelled

How much work does it take to lift an object of 10 kg

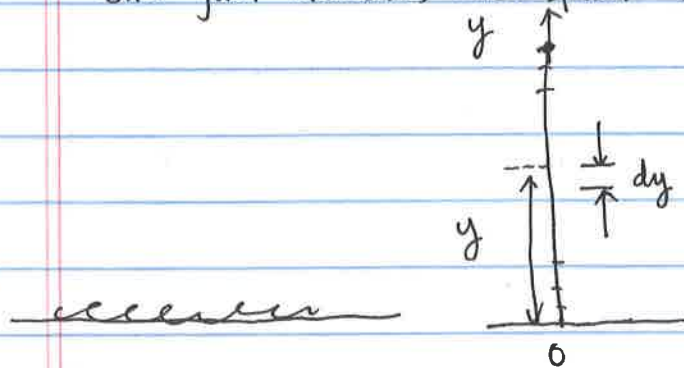
up by 1 m?

$$98 \text{ N} \cdot 1 \text{ m} = 98 \text{ N} \cdot \text{m} = 98 \text{ J}$$

(Answer)

2/22 (2)

Ex 2 A chain of weight 10kg and length 2m lies on the ground. How much work does it take to lift it ~~so~~ so that the lower end just touches the floor?



length weight
2 (m) 98 (N)
dy 1 ~~dy~~
Guess #1)
 $\frac{2}{dy} \cdot 98$

$$\int \text{work done to each } dy\text{-piece}$$
$$= \int (\text{weight of the } dy\text{-piece}) \cdot y$$
$$= \int_0^2 \frac{98 \cdot dy}{2} \cdot y$$

Guess #2)
 $\frac{dy}{2}$

$$= \frac{49}{2} \cdot \frac{y^2}{2} \Big|_0^2$$
$$= 49 \cdot \frac{2^2}{2 \cdot 2} = 49 \text{ J}$$

Ex 5: A chain of weight w and length $2m$ lies on the ground. How much work does it take to lift it so that the lower end just touches the floor?

length weight
 (m) $2m$ (m) w
 (mass #1) $\frac{w}{g} \cdot 2m$
 (mass #2) $\frac{w}{g} \cdot m$



work done to each piece

$$F \cdot (\text{weight of the piece}) =$$

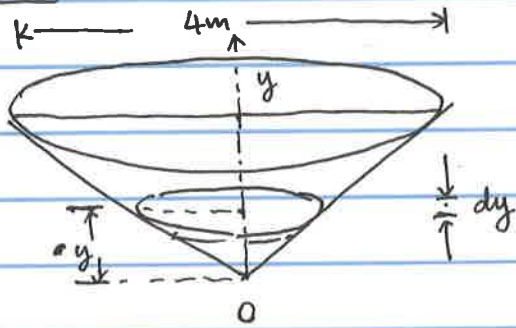
$$w \cdot \frac{w \cdot 2m}{g}$$

$$\frac{w}{g} \cdot \frac{2m}{g} =$$

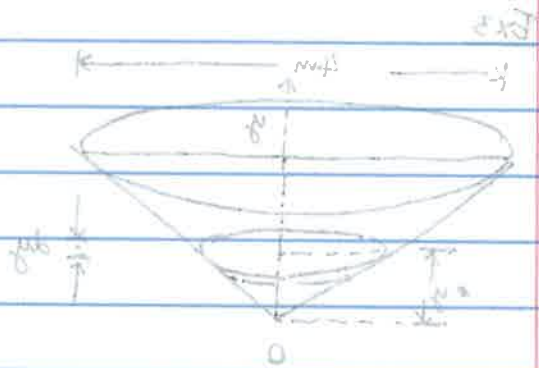
$$\frac{2w^2 m}{g^2}$$

2/22 (3)

Ex 3

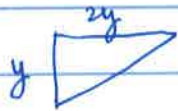
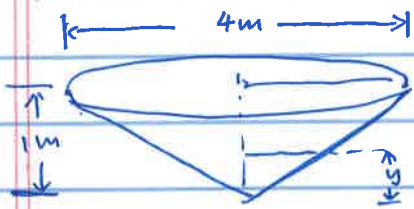


(3) 15/5



Calc II 2/26 ①

Ex1 Find the work to fill a ~~box~~ tank with water using the density of water 10^3 kg/m^3 , (1 g/cm^3)



slope of hypotenuse = $\frac{1}{2}$

$$r = 2y$$

distance traveled,
gravity of the disc $\int dy$

$$= \int \frac{\text{Volume}}{\text{mass}} \cdot \text{gravity} \cdot dy$$

$$= \int \pi r^2 \cdot dy \cdot 10^3 \cdot 9.8 \cdot y \, dy$$

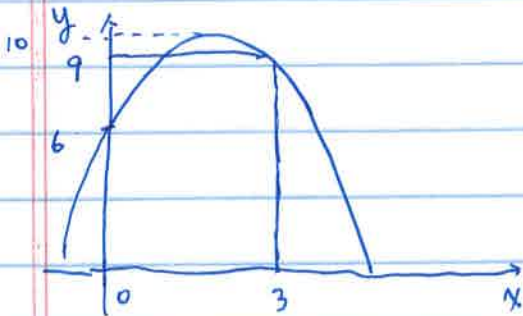
$$= \int \pi (2y)^2 dy \cdot 10^3 \cdot 9.8 \cdot y$$

$$= \pi \int_0^1 4y^3 dy \cdot 9.8 \cdot 10^3$$

$$= \pi \cdot \frac{4y^4}{4} \cdot 9.8 \cdot 10^3 \Big|_0^1$$

$$= 9800 \pi \text{ (J)}$$

Ex2 Find the average of the function $y = -x^2 + 4x + 6$ over $[0, 3]$



Q.10. Find the work to fill a tank with water using the amount of

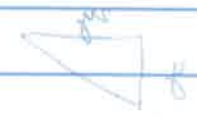
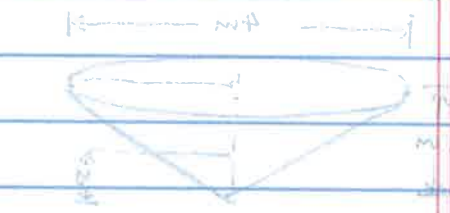
water is 10^3 kg/m^3 , (1 g/cm³)

quantity of

quantity of the disc

$$\int_0^4 \pi \cdot 8 \cdot 8 \cdot 10^3 \cdot (4-y) dy =$$

Volume
water
quantity



radius of bottom = 8

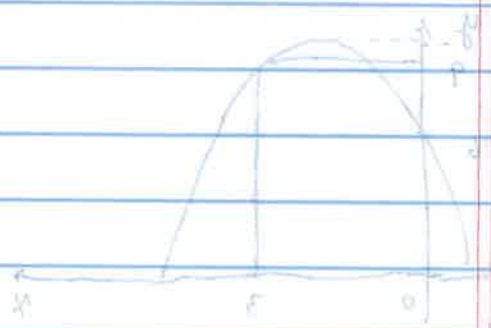
$$\int_0^4 \pi \cdot 8 \cdot 8 \cdot 10^3 \cdot (4-y) dy =$$

$$\int_0^4 \pi \cdot 64 \cdot 10^3 \cdot (4-y) dy =$$

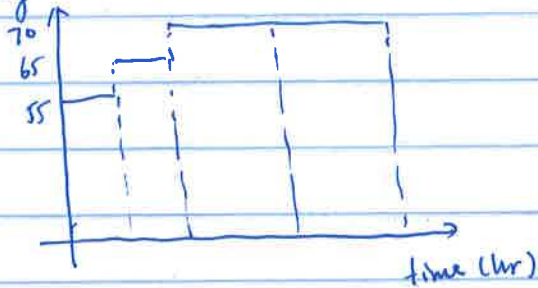
$$\int_0^4 \pi \cdot 64 \cdot 10^3 \cdot (4-y) dy =$$

$$= 800 \pi (1)$$

Find the average of the function $f(x) = x^2 + 1$ over [0, 3]



2/26 ②
Velocity (mph)



$$\text{distance} = 55 \cdot 0.5 + 65 \cdot 0.5 + 70 \cdot 2$$

distance = area under the graph of velocity

$$\text{average} = \frac{\text{distance}}{\text{time}} = \frac{\int \text{graph velocity}}{\text{time}}$$

Ex2: average = $\frac{\int_0^3 (-x^2 + 4x + 6) dx}{3}$

$$= \frac{-\frac{x^3}{3} + \frac{4x^2}{2} + 6x \Big|_0^3}{3}$$

$$= \frac{-9 + 18 + 18}{3} = \frac{27}{3} = 9$$

line at the
Find the intersection of the "average height" & the graph

$$-x^2 + 4x + 6 = 9$$

$$x = 1 \text{ and } x = 3$$

↗
You could always find such intersections.

③ $\frac{dV}{dt}$



(cm) unit

$$V = 20 \cdot 22 \cdot 10 = 44000 \text{ cm}^3$$

distance for depth with volume zero = constant

$$\frac{\text{distance}}{\text{time}} = \frac{\text{volume}}{\text{time}} = \text{speed}$$

$$\frac{44000}{t} = \text{speed}$$

$$44000 = \text{speed} \cdot t$$

$$P = \frac{F}{A} = \frac{mg}{A} = \frac{m \cdot g}{A}$$

unit area

Find the intersection of the graph height of the graph

$$P = 20 \cdot 22 \cdot 10 = 44000$$

$$E = 20 \text{ and } 22 \text{ and } 10$$

too early always find into intersection