HW 10

April 29, 2016

Problem 1. Evaluate the improper integral

$$\int_1^\infty \frac{1}{x^{\frac{3}{2}}} \,\mathrm{d}x \tag{1}$$

and state whether it converges.

Problem 2. Observe the calculation for problem 1, evaluate the improper integral

$$\int_1^\infty \frac{1}{x^p} \,\mathrm{d}x \tag{2}$$

for some given constant $p > 1$, then state whether it converges or diverges.

Problem 3. Evaluate the improper integral

$$\int_0^1 \frac{1}{x^p} \,\mathrm{d}x \tag{3}$$

for some given constant $p < 1$, then state whether it converges or diverges.

Problem 4. Evaluate

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \tan x \,\mathrm{d}x \tag{4}$$

and state whether it converges (we've done this problem in class.)

Problem 5. We mentioned the **Nivitanont Conjecture** in class:

Conjecture 1. *Let f be an elementary continuous function on $\mathbb{R}$ that is symmetric about the origin, then $\int_{-\infty}^{\infty} f(x) \,\mathrm{d}x$ is divergent.*

Here is how you can come up with a counterexample:
1) Show that

$$f(x) = e^{-|x|}x \tag{5}$$

has a graph that is symmetric about the origin. It is sufficient if you can show that $f(x) = -f(-x)$.

2) On $[0, \infty)$, $f(x)$ simplifies down to $f(x) = e^{-x}x$. Show that

$$\int_0^{\infty} e^{-x} x \, \mathrm{d}x \tag{6}$$

is convergent and find the improper integral (this integral is a member of a family called **Gamma functions**.)

The homework is now closed. It is due Monday, May 2rd.