11.10: Additional Problems Pt. 2

1. Find the Taylor series for $\sin x \cos x$.

2. Find the Taylor series for $\dfrac{e^x}{\ln(1+x)}$.

3. Compute the first 5 terms of the Taylor series for $\sqrt[3]{(2+x)^2}$.

Remainders

If we add the first n terms of a Taylor series, the remainder is:

$$|R_n(x)| \leq \frac{M}{(n+1)!}|x-a|^{n+1} \text{ for } |x-a| \leq d$$

where $|f^{(n+1)}(x)| \leq M$ for $|x-a| \leq d$

This measures the maximum in the error for stopping at only n terms.

Exercises

1. Find a bound on the remainder if you compute the Taylor series $f(x) = e^x$ out to 5 terms at $x = 0.$ and $x = 1$

2. Find a bound on the remainder if you compute the Taylor series $f(x) = \sin x$ out to 12 terms at any x.

Review: Limit Comparison Test

Recall that if

$$\lim_{n\to\infty} \frac{a_n}{b_n} = c$$

where c is finite and positive, then $\sum_{n=1}^{\infty} a_n$ converges or diverges whenever $\sum_{n=1}^{\infty} b_n$ converges or diverges.

To build intuition: The idea is that if $\lim_{n\to\infty} \frac{a_n}{b_n} = 3$, for example, then a_n is *roughly* $3 \cdot b_n$ for really really large n. So if $\sum b_n$ converges, $\sum a_n$ should also converge since, towards the end of the sum, the sum will be about three times the sum of b_n's.

The goal for these problems is to find another series that the above limit is finite.

- Determine whether $\sum_{n=1}^{\infty} \frac{1}{2^n - 1}$ converges or diverges.

- Determine whether $\sum_{n=1}^{\infty} \frac{2n^2 + 3n}{\sqrt{5 + n^5}}$ converges or diverges.