Introduction to Integrals

Express the limit as a definite integral on the given interval.

- $\lim_{n\to\infty} \sum_{i=1}^{n} \left[5(x_i^*)^3 - 4x_i^*\right] \Delta x, \; [2, 7]$

Use the form of the definition of the integral given in Theorem below to evaluate the integral.

- $\int_2^5 (4 - 2x)\; dx$

Theorem

$$\int_a^b f(x)dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$$

where $\Delta x = \frac{b-a}{n}$ and $x_i = a + i\Delta x$

Evaluate the integral by interpreting it in terms of areas.

- $\int_{-1}^{2} (1 - x)\ dx$

- $\int_{-1}^{2} |x|\ dx$

Properties of the Integral

1. $\int_a^b c\ dx = c(b - a)$, where c is any constant.
2. $\int_a^b [f(g) + g(x)]\ dx = \int_a^b f(x)dx + \int_a^b g(x)\ dx.$
3. $\int_a^b cf(x)\ dx = c\int_a^b f(x)\ dx$, where c is any constant.
4. $\int_a^b [f(x) - g(x)]\ dx = \int_a^b f(x)\ dx - \int_a^b g(x)\ dx$

- Evaluate $\int_{\pi}^{\pi} \sin^2 x \cos^4 x \, dx$.

- If $\int_0^1 x^2 \, dx = \frac{1}{3}$, evaluate $\int_0^1 (5 - 6x^2) \, dx$.

- Write as a single integral in the form $\int_a^b f(x) \, dx$:

$$\int_{-2}^{2} f(x) \, dx + \int_{2}^{5} f(x) \, dx - \int_{-2}^{-1} f(x) \, dx$$

- If $\int_0^9 f(x)\ dx = 37$ and $\int_0^9 g(x)\ dx = 16$, find $\int_0^9 [2f(x) + 3g(x)]\ dx$.

- Use Property 3 below to estimate the value of the integral:

$$\int_0^2 \frac{1}{1+x^2}\ dx$$

Comparison Properties

1. If $f(x) \geq 0$ for $a \leq x \leq b$, then $\int_a^b f(x)\ dx \geq 0$.
2. If $f(x) \geq g(x)$ for $a \leq x \leq b$, then $\int_a^b f(x)\ dx \geq \int_a^b g(x)\ dx$
3. If $m \leq f(x) \leq M$ for $a \leq x \leq b$, then $m(b-a) \leq \int_a^b f(x)\ dx \leq M(b-a)$