- If $\int_0^9 f(x)\,dx = 37$ and $\int_0^9 g(x)\,dx = 16$, find $\int_0^9 [2f(x) + 3g(x)]\,dx$.

$$\int_0^9 [2f(x)+3g(x)]\,dx$$
$$= 2\int_0^9 f(x)\,dx + 3\int_0^9 g(x)\,dx$$
$$= 2(37) + 3(16)$$
$$= 74 + 48 = \boxed{122}$$

- Use Property 3 below to estimate the value of the integral:

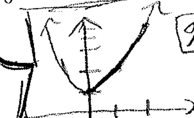
$$\int_0^2 \frac{1}{1+x^2}\,dx$$

From our Bounds $\to 0 \le x \le 2$

$$1+0^2 \le 1+x^2 \le 1+(2)^2$$
$$1 \le 1+x^2 \le 5$$
$$1 \ge \frac{1}{1+x^2} \ge \frac{1}{5}$$

Property 3 below $\to$ $1(2-0) \ge \int_0^2 \frac{1}{1+x^2}\,dx \ge \frac{1}{5}(2-0)$

$$2 \ge \int_0^2 \frac{1}{1+x^2}\,dx \ge \frac{2}{5}$$



graph of $1+x^2$

because the graph is increasing from $x=0$ to $x=2$, our inequalities do not switch.

graph of $\frac{1}{x}$

because the graph is decreasing from $x=1$ to $x=5$, we switch the direction of our inequalities.

Comparison Properties

1. If $f(x) \ge 0$ for $a \le x \le b$, then $\int_a^b f(x)\,dx \ge 0$.

2. If $f(x) \ge g(x)$ for $a \le x \le b$, then $\int_a^b f(x)\,dx \ge \int_a^b g(x)\,dx$

3. If $m \le f(x) \le M$ for $a \le x \le b$, then $m(b-a) \le \int_a^b f(x)\,dx \le M(b-a)$