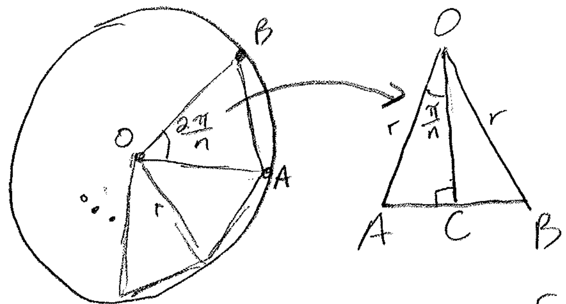
1. Let A_n be the area of a polygon with n equal sides inscribed in a circle with radius r. By dividing the polygon into n congruent triangles with central angle $\frac{2\pi}{n}$, show that:

$$A_n = \frac{1}{2}nr^2\sin\left(\frac{2\pi}{n}\right)$$



$|AC| = r\sin\frac{\pi}{n}$

$|OC| = r\cos\frac{\pi}{n}$

$|AB| = 2r\sin\frac{\pi}{n}$

Area of $\Delta ABO = \frac{1}{2}bh$

$= \frac{1}{2}|AB|\cdot|CO|$

$= \frac{1}{2}(2r\sin\frac{\pi}{n})(r\cos\frac{\pi}{n})$

$= r^2\sin\frac{\pi}{n}\cos\frac{\pi}{n}$

(Note: $2\sin x\cos x = \sin 2x$

$= \frac{1}{2}r^2\sin(\frac{2\pi}{n})$

Area of the polygon $= \frac{1}{2}nr^2\sin(\frac{2\pi}{n})$

2. Show that $\lim_{n\to\infty} A_n = \pi r^2$

Recall from Calculus I: $\lim_{\theta\to 0}\frac{\sin\theta}{\theta} = 1$.

$\lim_{n\to\infty} \frac{1}{2}nr^2\sin(\frac{2\pi}{n})$

$= \lim_{n\to\infty} \frac{1}{2}r^2\frac{\sin(\frac{2\pi}{n})}{\frac{1}{n}}$

$= \lim_{n\to\infty} \frac{1}{2}r^2\frac{2\pi\sin(\frac{2\pi}{n})}{\frac{2\pi}{n}} = \lim_{n\to\infty} \pi r^2\frac{\sin(\frac{2\pi}{n})}{\frac{2\pi}{n}}$

$= \frac{1}{2}r^2(2\pi) = \pi r^2$