Implicit Differentiation

1. $16x^2 + 25y^2 = 400$

2. $x\sin(y^2) = 1$

3. $\sqrt{x} + \sqrt{y} = 1$

4. $x^2 + xy + y^2 = 9$

5. $3x^2y + 2xy^3 = 1$

6. $(x-1)y^2 = x + 1$

7. $\frac{1}{x+1} + \frac{1}{y+1} = 1$

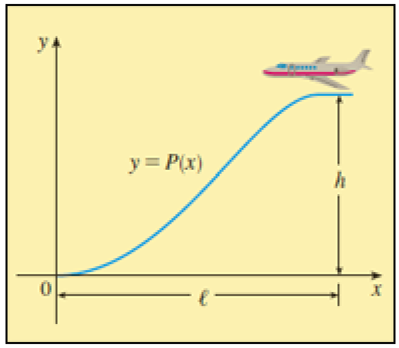
8. $xy^5 + x^5y = 1$

Find y''

1. $xy = 1$

2. $x^2 + y^2 = 4$

Project: Where Should A Pilot Start Descent? [1]



An approach path for an aircraft landing is shown in the figure and satisfies the following conditions:

(i) The cruising altitude is h when descent starts at a horizontal distance l from touchdown at the origin.

(ii) The pilot must maintain a constant horizontal speed v throughout descent.

(iii) The absolute value of the vertical acceleration should not exceed a constant k (which is much less than the acceleration due to gravity).

1. Find the cubic polynomial $P(x) = ax^3 + bx^2 + cx + d$ that satisfies condition (i) by imposing suitable conditions on $P(x)$ and $P'(x)$ at the start of descent and at touchdown.

[1] *Calculus*, Stewart

2. Use conditions (ii) and (iii) to show that:

$$\frac{6hv^2}{l^2} \leq k$$

3. Suppose that an airline decides not to allow vertical acceleration of a plane to exceed $k = 860$ mi/h^2. If the cruising altitude of a plane is 35,000 ft and the speed is 300 mi/h, how far away from the airport should the pilot start to decend? Note: You are now able to graph the path of decent.

Challenge Problems

These problems are difficult!

1. Find the values of the constants a and b such that

$$\lim_{x \to 0} \frac{\sqrt[3]{ax+b} - 2}{x} = \frac{5}{12}$$

2. Show that

$$\frac{d}{dx}\left(\frac{\sin^2 x}{1+\cot\, x} + \frac{\cos^2 x}{1+\tan\, x}\right) = -\cos\, 2x$$

3. If f is differentiable at a, where $a > 0$, evaulate the following limit in terms of $f'(a)$

$$\lim_{x \to a} \frac{f(x) - f(a)}{\sqrt{x} - \sqrt{a}}$$

4. Evaluate $\displaystyle\lim_{x \to 0} \frac{\sin(3+x)^2 - \sin 9}{x}$.

1	3						4	
			5		6	9		
			7		1	2	3	
	5	6						
		2	8	6	4	5		
						8	2	
	9	4	6		7			
		8	4		3			
	6						8	2

	1	3	5		6			
	6	2			8			
								1
1				9	5		3	
4				1				6
	2		4	8				5
2								
			7			4	6	
			2		4	7	1	