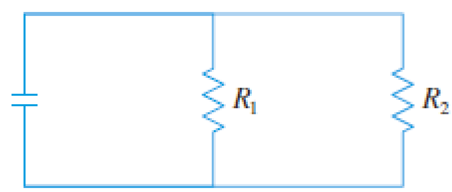
Related Rates

1. If a snowball melts so that its surface area decreases at a rate of 1 cm^2/min, find the rate at which the diameter decreases when the diameter is 10 cm.

2. If two resistors with resistances R_1 and R_2 are connected in parallel, as in the figure, then the total resistance R, measured in ohms (Ω), is given by

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

   If R_1 and R_2 are increasing at rates of 0.3 Ω/s and 0.2 Ω/s, respectively, how fast is R changing when $R_1 = 80\ \Omega$ and $R_2 = 100\ \Omega$?



3. The minute hand on a watch is 8 mm long and the hour hand is 4 mm long. How fast is the distance between the tips changing at one o'clock?

4. When air expands adiabatically (without gaining or losing heat), its pressure P and volume V are related by the equation $PV^{1.4} = C$, where C is a constant. Suppose that a certain instant the volume is 400 cm^3 and the pressure is 80 kPa and is decreasing at a rate of 10 kPa/min. At what rate is the volume increasing in this instant?

5. A Ferris wheel with a radius of 10 m is rotating at a rate of one revolution every 2 minutes. How fast is a rider rising when his seat is 16 m above ground level?

6. A particle moves along the curve $y = 2\sin(\frac{\pi x}{2})$. As the particle passes through the point $(\frac{1}{3}, 1)$, its x-coordinate increases at a rate of $\sqrt{10}$ cm/s. How fast is the distance from the particle to the origin changing at this instant?

Review

Calculate the following **using the definition of the derivative**:

1. $y = \frac{1}{2}x - \frac{1}{3}$
2. $y = \frac{1-2t}{3+t}$
3. $y = x^4$
4. $y = \sqrt{3+x}$

Calculate the following derivatives:

1. $f(x) = 2^{40}$
2. $f(x) = x^3 - 4x + 6$
3. $y = x^2(1-2x)$
4. $y = x^{\frac{5}{3}} - x^{\frac{2}{3}}$
5. $f(t) = \sqrt{t} - t$
6. $f(x) = (\frac{1}{x^2} - \frac{3}{x^4})(x + 5x^3)$
7. $y = \frac{1+2x}{3-4x}$
8. $f(x) = \frac{x^2+4x+3}{\sqrt{x}}$

9. $f(x) = \sin x + \frac{1}{2}\cot x$

10. $y = 4\sec x - \csc x$

11. $y = \frac{\cos x}{1 - \sin x}$

12. $f(t) = (3t-1)^4(2t+3)^{-3}$

13. $f(x) = (\frac{x}{x^3+1})^6$

14. $y = \cos\sqrt{\sin(\tan \pi x)}$

A runner sprints around a circular track of radius 100 m at a constant speed of 7 m/s. The runner's friend is standing at a distance 200 m from the center of the track. How fast is the distance between the friends changing when the distance between them is 200 m?

Find the linearization $L(x)$ at a.

1. $f(x) = x^4 + 3x^2$, $a = -1$.

2. $f(x) = \sqrt{x}$, $a = 4$.

Find the differential of the following functions.

1. $y = u\cos u$.

2. $y = \sqrt{z + \frac{1}{z}}$.

Use linear approximation to estimate the given number.

1. $\frac{1}{4.002}$.

2. $\sqrt{99.8}$.

3. $\sin 1°$.

Use differentials to estimate the given number.

1. $\frac{1}{4.002}$.

2. $\sqrt{99.8}$.

3. $\sin 1°$.


Happy Pi Day
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