Continuity

Are the following functions continuous at a?

1. $f(x) = \begin{cases} 1 - x^2 & x < 1 \\ \frac{1}{x} & x \geq 1 \end{cases}, \quad a = 1$

2. $\begin{cases} cosx & x < 0 \\ 0 & x = 0, \\ 1 - x^2 & x > 0 \end{cases} \quad a = 0$

Where are the following discontinuous?

1. $\begin{cases} x + 2 & x < 0 \\ 2x^2 & 0 \leq x \leq 1 \\ 2 - x & x > 1 \end{cases}$

2. $\begin{cases} x + 1 & x \leq 1 \\ \frac{1}{x} & 1 < x < 3 \\ \sqrt{x - 3} & x \geq 3 \end{cases}$

Intermediate Value Theorem

Show the following have a root in the given interval.

1. $\sqrt[3]{x} = 1 - x$, $(0, 1)$

2. $\sin x = x^2 - x$, $(1, 2)$

Review

Find the following limits:

1. $\lim_{x\to 0} \cos(x + \sin x)$
2. $\lim_{x\to -3} \dfrac{x^2 - 9}{x^2 + 2x - 3}$
3. $\lim_{x\to 1^+} \dfrac{x^2 - 9}{x^2 + 2x - 3}$
4. $\lim_{x\to 2} \dfrac{x^2 - 4}{x^3 - 8}$
5. $\lim_{x\to 4^+} \dfrac{4 - x}{|4 - x|}$
6. $\lim_{x\to 16} \dfrac{4 - \sqrt{x}}{s - 16}$
7. $\lim_{h\to 0} \dfrac{(h-1)^3 + 1}{h}$
8. $\lim_{x\to 0} \left(\dfrac{1}{x-1} + \dfrac{1}{x^2 - 3x + 2} \right)$

Prove that $\lim_{x\to 0} x^2 \cos(\frac{1}{x^2}) = 0$.

Use the Intermediate Value Theorem to show that there is a root of the equation in the given interval.

1. $x^5 - x^3 + 3x - 5 = 0$, $(1, 2)$
2. $2\sin x = 3 - 2x$, $(0, 1)$