Sketching a Curve

Sketch the following curves using elements of Calculus.

1. $y = x^3 - 12x^2 + 36x$
2. $y = x^5 - 5x$
3. $y = \frac{x-1}{x^2}$
4. $y = \frac{x}{x^3-1}$
5. $y = \frac{x^2}{x^2+3}$
6. $y = (x-3)\sqrt{x}$
7. $y = \sqrt{x^2 + x - 2}$
8. $y = \frac{x}{\sqrt{x^2-1}}$

Challenge Problems

1. Show that $|\sin x - \cos x| \leq \sqrt{2}$ for all x.

2. Show that $x^2y^2(4 - x^2)(4 - y^2) \leq 16$ for all numbers x and y such that $|x| \leq 2$ and $|y| \leq 2$.

3. Find the highest and lowest points on the curve $x^2 + xy + y^2 = 12$.

4. Find a function f such that $f'(-1) = \frac{1}{2}$, $f'(0) = 0$, and $f''(x) > 0$ for all x, or prove that such a function cannot exist.

Quiz 3 Problems

Complete these problems by April 10th. Show your work.

1. Given the graph $y = \frac{6}{x^2+3}$, find the tangent line with maximum slope and the tangent line of minimum slope.

2. Given two non-negative numbers who sum to 9, find the maximum of the product of one number and the square of the other.