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The Topology of Direct Limits of Direct Systems Induced by Maps

Abstract. It is an unfortunate fact that the direct limit of a direct system of spaces, even under seemingly ideal conditions, need not have certain desirable topological properties. The second author confronted this situation in his study of the existence of universal objects in the theory of cohomological dimension. For example, even if the system consists of compact Hausdorff spaces and injective maps (i.e., embeddings), its direct limit need not be normal. It is not even clear if its limit would be regular. It is almost impossible for its limit to be compact, so how close could it come to being compact and would such a "near compactness" property be useful?

We will show in this seminar, based on the research of the two named authors, how to construct such a direct system in a certain easily understood way, will describe its direct limit X_δ, and will explain how to obtain the topology of X_δ. Then we will discuss the topological properties that can be expected from this direct limit as well as those that cannot ensue from this method. Our final remarks will concern a more abstract method of constructing a direct system whose direct limit could exhibit the important properties that are needed in the study of cohomological dimension and which are lacking in the previously mentioned models.