SUPPLEMENTAL EXERCISES FOR CH. 15, §7 AND §8

1. Exercises

(1) Calculate the volume of the solid E in $\mathbb{R}^3$ that lies: to the left of the plane $y = 0$; below the cone $z = \sqrt{x^2 + y^2}$; above the cone $z = -\sqrt{x^2 + y^2}$; and inside the sphere $x^2 + y^2 + z^2 = 9$.

(2) Calculate the volume of the solid E in $\mathbb{R}^3$ that lies: to the right of the plane $y = 0$; behind the plane $y = x$; in front of the plane $y = -x$; above the plane $z = -2$; below the plane $z = 1 - x$; and inside the cylinder $x^2 + y^2 = 1$.

(3) Evaluate

$$\iiint_E y^2 \, \mathrm{d}V$$

where E is the solid in $\mathbb{R}^3$ that lies: inside the sphere $x^2 + y^2 + z^2 = 4$; outside the sphere $x^2 + y^2 + z^2 = 1$; above the cone $z = \sqrt{x^2 + y^2}$; and in front of the plane $y = -x$.

(4) Evaluate

$$\iiint_E y e^{x^2+y^2+z^2} \, \mathrm{d}V$$

where E is the solid in $\mathbb{R}^3$ that lies: above the cone $z = \sqrt{x^2 + y^2}$; below the cone $z = \sqrt{3x^2 + 3y^2}$; inside the sphere $x^2 + y^2 + z^2 = 4$; behind the plane $x = 0$; and to the right of the plane $y = 0$.

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2. Solutions

(1) $9\pi\sqrt{2}$

(2) $\dfrac{3\pi}{4}$

(3) $\displaystyle\int_0^{\pi/4}\int_{-\pi/4}^{3\pi/4}\int_1^2 \rho^4\sin^3(\phi)\sin^2(\theta)\,\mathrm{d}\rho\,\mathrm{d}\theta\,\mathrm{d}\phi = \frac{31\pi}{120}(8-5\sqrt{2})$

(4) $\displaystyle\int_{\pi/6}^{\pi/4}\int_{\pi/2}^{\pi}\int_0^2 e^{\rho^2}\rho^3\sin^2(\phi)\sin(\theta)\,\mathrm{d}\rho\,\mathrm{d}\theta\,\mathrm{d}\phi = \frac{1}{48}(1+3e^4)(-6+3\sqrt{3}+\pi)$. Note: the integration step is particularly involved here.