MATH 2934 Problem Types List

Julia Jackson

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Contents

Introduction

On the following pages you will find a list of problem types from each section of the textbook. These are the problem types covered in the homework, and are the types you should know how to solve in order to be prepared for the exams.

I hope that you will find this document useful as a study guide.

If you have any questions on how to approach any of these problem types, please don't hesitate to come to my office hours, ask questions during discussion or on Canvas, or send me an e-mail. I'm more than happy to provide any guidance you might need!

I wish you the best of luck as you prepare for your exams!

Chapter 12

12.5: Equations of Lines and Planes

- Give vector, parametric, and/or symmetric equations for a line:
  - Through a given pair of points
  - Through a given point and parallel to a given vector
  - Through a given point and parallel to a given line
  - Through a given point and perpendicular to a given plane
  - Of intersection of two planes
- Determine whether a given pair of lines are parallel, perpendicular, intersecting, or skew
- Give vector, scalar, and/or linear equations for a plane:
  - Through three given points
  - Through a given point and perpendicular to a given vector
  - Through a given point and perpendicular to a given line
  - Through a given point and parallel to a second plane
  - Containing a given line and a given point
  - Containing a given pair of intersecting lines
- Find the point at which a given line intersects a given plane
- Calculate the angle between two given planes

12.6: Cylinders and Quadratic Surfaces

- Identify whether a given surface is a plane, a sphere, a cylinder, or one of the six quadratic surfaces given on our chart of quadratic surfaces.
  - Sketch said surface
  - You may need to put a quadratic surface into standard form first.
- Use traces to sketch a given surface.
- Match the equation of a surface to its graph
- Given a set of traces, sketch a surface that has those traces

Chapter 13

13.1: Vector Functions and Space Curves

- Give the domain of a given vector function
- Evaluate the limit of a given vector function
  - You may or may not need L'Hopital's rule.
- Sketch the graph of a given vector function
- Match a given vector function (or a given set of parametric equations corresponding to a vector function) with its graph.
- Give the t-value(s) at which a given vector function passes through a given point.

13.2: Derivatives and Integrals of Vector Functions

- Given a vector function, calculate its derivative:
  - In general
  - At a given t-value
  - At a given point on its graph
- Given a vector function $\overrightarrow{r}(t)$, give a vector that lies tangent to the graph of $\overrightarrow{r}(t)$:
  - At a given t-value
  - At a given point on the graph
  - You may also be asked to give a *unit* tangent vector, instead
- Given a vector function $\overrightarrow{r}(t)$, give a line that lies tangent to the graph of $\overrightarrow{r}(t)$:
  - At a given t-value
  - At a given point on the graph
- Evaluate a definite integral of a vector function
- Evaluate an indefinite integral of a vector function
- Given the derivative $\overrightarrow{r}'(t)$ of a vector function $\overrightarrow{r}(t)$ and an initial value of $\overrightarrow{r}(t)$ (i.e. $\overrightarrow{r}(k)$ for some constant k), calculate $\overrightarrow{r}(t)$ for any value of t.

13.3: Arc Length

- Calculate the length, on the graph of a given vector function $\vec{r}(t)$, of an arc that lies:
  - Between two t-values
  - Between two points on the graph
- Given a vector function $\vec{r}(t)$ and a point P on the graph of $\vec{r}(t)$, calculate:
  - The arc length function $s(t)$ for $\vec{r}(t)$ measured from P in the direction of increasing t; then
  - Use it to reparametrise $\vec{r}(t)$ with respect to this arc length function; and finally
  - Find a point L units away from P along the graph of $\vec{r}(t)$ in the direction of increasing t.

13.4: Motion in Space: Velocity and Acceleration

- Given the position $\vec{r}(t)$ of an object in space, calculate its:
  - Velocity, Acceleration, and/or Speed
    * In general
    * At a given t-value
    * At a given point along its path
- Given an object's acceleration $\vec{a}(t)$ and initial velocity, calculate its velocity at any value of t
- Given an object's velocity $\vec{v}(t)$ and initial position, calculate its position at any value of t
- Given an object's acceleration $\vec{a}(t)$, initial velocity, and initial position, calculate its position at any value of t

Chapter 14

14.1: Functions of Several Variables

- Find and sketch the domain of a given two-variable function.
- Sketch the graph of a given two-variable function by:
  - Recognizing its graph as a surface we learned to graph in Chapter 12; or
  - Sketching a few level curves and stitching them together in $\mathbb{R}^3$
- Use a given contour map to estimate the output of a two-variable function
- Use a given contour map of a function to sketch a possible graph of that function
- Create a contour map of a given function
- Match a given function with its graph

14.2: Limits and Continuity

- Evaluate the limit of a given function at a given point using continuity (i.e. by plugging in)
- Show that the limit of a given function at a given point does not exist by showing that its limits along different paths do not match
- Evaluate the limit of a given function at a given point using conjugation
- Evaluate the limit of a given function at a given point using the squeeze theorem

14.3: Partial Derivatives

- Estimate the partial derivatives of a given function at a given point using only a given graph of the function.
- Match a graph of a given function with the graphs of its partial derivatives.
- Calculate the first partial derivatives of a given function:
  - Of two or more variables;
  - At a general point (e.g. (x, y)), or at a given point (e.g. (a, b)).
  - These functions could be: polynomials; exponential functions; logarithmic functions; trigonometric functions; nth-root functions; or any sum, difference, product, quotient, or composition of these.
- Use implicit differentiation to compute a given partial derivative
  - Know how to recognize when implicit differentiation needs to be used.
- Evaluate higher partial derivatives of a given function:
  - Of two or more variables;
  - At a general point (e.g. (x, y)), or at a given point (e.g. (a, b)).
  - Know how to use Clairaut's Theorem to save some work on such problems.

14.4: Tangent Planes and Linear Approximations

- Give an equation of a plane that lies tangent to a given surface $z = f(x, y)$ at a given point.
- Give an equation of a plane that lies tangent to the graph of a given function at a given point $(a, b, f(a, b))$.
- Give the linearization of a given function $f(x, y)$ at a given point (a, b).
  - Use this linearization to estimate the value of $f(x, y)$ at a given point near (a, b).

14.5: The Chain Rule

- Use the chain rule to evaluate the (ordinary or partial, depending on context) derivative(s) of a multivariable function $f(x_1, x_2, \ldots, x_m)$ when each x_i is itself a function $x_i(t_1, t_2, \ldots, t_n)$:
  - Either in general or at a specific point $(t_1, t_2, \ldots, t_n) = (c_1, c_2, \ldots, c_n)$.
  - Pay particular attention to which derivatives are partial and which are ordinary.

14.6: Directional Derivatives and the Gradient Vector

- Calculate the derivative of a given function f at a given point in the direction of a given vector $\vec{v}$, where $\vec{v}$ may or may not be a unit vector.
  - Know the formula for this derivative that uses the gradient vector, as it applies to a function of any number of variables.
  - That is, know how to do this no matter how many variables f has.
  - The function could be a: polynomial; exponential function; logarithmic function; trigonometric function; nth-root function; or any sum, difference, product, quotient, or composition of these.
- Calculate the derivative of a given function f at a given point in the direction of another given point.
- Use the gradient vector to calculate the maximum instantaneous rate of change of a given function at a given point, and give the direction in which this instantaneous rate of change occurs.
- Use the gradient vector to give an equation of the plane that lies tangent to a given surface $F(x, y, z) = k$ (for k a constant) at a given point (a, b, c).
- Use the gradient vector to give an equation of the line that is orthogonal to a given surface $F(x, y, z) = k$ (for k a constant) at a given point (a, b, c).

14.7: Maximum and Minimum Values

- Find the critical points of a given function $f(x, y)$.
  - Classify each as either a saddle point, or corresponding to a local maximum or local minimum value of f using the Second Derivative Test.
  - The function could be a: polynomial; exponential function; logarithmic function; trigonometric function; nth-root function; or any sum, difference, product, quotient, or composition of these.
- Find the absolute minimum and maximum values of a given function $f(x, y)$ on a closed subset D of its domain.
  - The region D may be triangular, rectangular, or circular.
- Solve a given optimization problem. That is, find the maximum and minimum values of a given function, subject to a given constraint (or restriction) on its variables.

14.8: Lagrange Multipliers

- Calculate the absolute maximum and minimum values of a given function $f(x, y)$ or $f(x, y, z)$ subject to a given constraint equation.
  - Such a question may be framed as an optimization problem.
- Calculate the absolute minimum and maximum values of a given function $f(x, y)$ on a closed subset D of its domain.
  - The method of Lagrange Multipliers is particularly useful when the boundary of D is circular.
  - See also: Section 14.7.

Chapter 15

15.1: Double Integrals Over Rectangles

- Estimate the double integral of a given two-variable function over a given rectangular subset of its domain using a Riemann sum.
  - Alternatively, estimate the total signed volume that lies between a given surface (or the graph of a given two-variable function) and a given rectangle in the xy-plane using a Riemann sum.
- Evaluate a given iterated integral.
- Evaluate the double integral of a given two-variable function over a given rectangular subset of its domain.
  - Alternatively, calculate the total signed volume that lies between a given surface (or the graph of a given two-variable function) and a given rectangle in the xy-plane.
- Calculate the average value of a given two-variable function over a given rectangular subset of its domain.

15.2: Double Integrals Over General Regions

- Evaluate the double integral of a given two-variable function over a given subset D of its domain.
  - D may be given as a set of points or as a region bounded by various curves in the xy-plane.
  - D may be type I, type II, both, or neither. If D is of both types, know how to set up and, if possible, evaluate the double integral in both orders of integration.
- Calculate the volume of a given solid in $\mathbb{R}^3$ using a double integral.
- Given an iterated integral, reverse the order of integration by first sketching the region of integration.
- Calculate the area inside a given subset D of $\mathbb{R}^2$.
  - Use this area to help calculate the average value of a given function f on D.

15.3: Double Integrals in Polar Coordinates

- Evaluate the double integral of a given two-variable function over a given subset D of its domain, where D is best parametrized with polar coordinates.

- Calculate the area inside a given subset D of $\mathbb{R}^2$, where D is best parametrized with polar coordinates.

  - Use this area to help calculate the average value of a given function f on D.

- Calculate the volume of a given solid in $\mathbb{R}^3$ using a double integral in polar coordinates.

15.5: Surface Area

- Calculate the area of a piece S of a surface.
  - The region in the xy-plane above or below S may be best described in either rectangular or polar coordinates. This same region may be given explicitly, or you may have to find it for yourself.
  - The surface that S is a part of may or may not be written explicitly as $z = f(x, y)$. Know how to handle each situation.

15.6: Triple Integrals

- Evaluate the triple integral of a given three-variable function over a given subset E of its domain.
  - E may be given as a set of points or as a region bounded by various surfaces in $\mathbb{R}^3$.
  - E may be type 1, type 2, type 3, all three, or none of these. If E is of all three types, know how to set up and, if possible, evaluate the triple integral in all six orders of integration.
- Calculate the volume of a given solid in $\mathbb{R}^3$ using a triple integral.

15.8: Triple Integrals in Spherical Coordinates

- Evaluate the triple integral of a given three-variable function over a given subset E of its domain, where E is best parametrized with spherical coordinates.

- Calculate the volume of a given solid E in $\mathbb{R}^3$ using a triple integral, where E is best parametrized with spherical coordinates.

Chapter 16

16.1: Vector Fields

- Match a given vector field with its graph.
- Match a given function with a graph of its gradient vector field.

16.2: Line Integrals

- Evaluate the line integral with respect to arc length of a given real-valued function f along a given curve C.
  - f may have two or three variables.
  - The parametrization of C may be given to you, or;
  - If such a parametrization is not given, C may be a line segment; an arc on a circle of radius R centered at the origin; or an arc on the graph of a function $y = f(x)$ or $x = f(y)$.
  - C may also be made up of several distinct pieces of these types.
  - Problems may ask you to evaluate such an integral directly, or they may ask you to calculate the total signed volume between the graph of f and the curve C.
- Evaluate the line integral with respect to x, y, or z of a given real-valued function f along a given curve C.
  - f may have two or three variables.
  - The parametrization of C may be given to you, or;
  - If such a parametrization is not given, C may be a line segment; an arc on a circle of radius R centered at the origin; or an arc on the graph of a function $y = f(x)$ or $x = f(y)$.
  - C may also be made up of several distinct pieces of these types.
  - Be aware of the effect that the orientation of C has on such an integral.
- Evaluate the line integral of a given vector field $\vec{F}$ along a given curve C.
  - $\vec{F}$ may have two or three variables.
  - The parametrization of C may be given to you, or;
  - If such a parametrization is not given, C may be a line segment; an arc on a circle of radius R centered at the origin; or an arc on the graph of a function $y = f(x)$ or $x = f(y)$.
  - C may also be made up of several distinct pieces of these types.

- Problems may ask you to evaluate such an integral directly, or they may ask you to calculate the total work done by a force field $\vec{F}$ as an object moves along the curve C.
- Be aware of the effect that the orientation of C has on such an integral.

16.3: The Fundamental Theorem for Line Integrals

- Determine whether a given vector field $\vec{F}(x, y)$ is conservative.
  - Know how to find a potential function $f(x, y)$ for $\vec{F}(x, y)$ if $\vec{F}$ is conservative.
- Given a conservative vector field $\vec{F}(x, y) = \big\langle P(x, y), Q(x, y)\big\rangle$, find a potential function $f(x, y)$ for $\vec{F}$ and use it to evaluate the line integral of $\vec{F}$ along a given curve C.
  - You may first need to determine that $\vec{F}$ is conservative.
  - You may only be given the endpoints of C and not C itself.
  - Problems may ask you to evaluate such an integral directly, or they may ask you to calculate the total work done by a force field $\vec{F}$ as an object moves either along the curve C, or between two given points in $\mathbb{R}^2$.
  - When stated directly, such a problem may ask you to evaluate either
    $$\int_C \vec{F} \cdot \mathrm{d}\vec{r}$$
    or
    $$\int_C P \,\mathrm{d}x + Q \,\mathrm{d}y$$
    Be aware of the equivalence of these forms.

16.4: Green's Theorem

- Given a vector field $\overrightarrow{F}(x,y) = \left\langle P(x,y), Q(x,y) \right\rangle$ and a piecewise-smooth, simple closed curve C in $\mathbb{R}^2$, use Green's Theorem to evaluate the integral of $\overrightarrow{F}$ along C.
  - C may be positively- or negatively-oriented. Know how to handle each case.
  - Problems may ask you to evaluate such an integral directly, or they may ask you to calculate the total work done by a force field $\overrightarrow{F}$ as an object moves either along the curve C, or between two given points in $\mathbb{R}^2$.
  - When stated directly, such a problem may ask you to evaluate either
    $$\int_C P \, \mathrm{d}x + Q \, \mathrm{d}y$$
    or
    $$\int_C \overrightarrow{F} \cdot \mathrm{d}\overrightarrow{r}$$
    Be aware of the equivalence of these forms.
  - The region D enclosed by the curve C may be best described with either rectangular or polar coordinates. If the former, D may be of Type I, Type II, both, or neither. Know how to handle each of these cases.