MATH 2934 FAQ

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Contents

1 Introduction

1.1 An Overview of the Course

Welcome to MATH 2934: Differential and Integral Calculus III! This course broadly consists of three major parts: the calculus of curves in $\mathbb{R}^3$ (Chapters 12 and 13 of our text); the calculus of surfaces in $\mathbb{R}^3$ (Chapters 14 and 15 of our text, and the bulk of the course); and the calculus of vector fields in $\mathbb{R}^2$ and $\mathbb{R}^3$ (Chapter 16 of our text). The following paragraphs outline the material in each chapter more specifically (words in bold signify vocabulary that we will introduce during the course).

Chapter 12 sets up the language of three-dimensional coordinate systems that pervades the course. You should have seen most of this material already (see the next section for more details). We will begin the course by finishing out this chapter. We will give several different equations that may be used to describe lines in $\mathbb{R}^3$ (equations which also work in $\mathbb{R}^2$), and we will introduce the core concept of **surfaces** in $\mathbb{R}^3$. In particular, we will give equations for planes, cylinders, and some simple **quadratic surfaces**.

Chapter 13 brings us back to calculus. We will introduce **space curves**, i.e. curves in $\mathbb{R}^3$, and learn how to parametrize them using **vector functions**. We will then discuss the derivatives and integrals of such functions.

Chapter 14 brings us to the calculus of surfaces in $\mathbb{R}^3$. We will formally introduce multivariable functions for the first time, and discuss how to find limits and derivatives (more accurately, **partial** and **directional derivatives**) of such functions. We will then introduce the key concept of the **gradient** (or more specifically, the **gradient vector field**) of a multivariable function. After all of this, we talk about applications of differentiation; in particular, we will look at optimization.

Chapter 15 finishes out the calculus of surfaces in $\mathbb{R}^3$. In it, we will discuss integration of functions of two and three variables. To facilitate this integration, we will need to introduce two new coordinate systems that can be used to describe points in $\mathbb{R}^3$: **cylindrical coordinates** and **spherical coordinates**. We will also discuss how to find the area of a section of a surface in $\mathbb{R}^3$.

Finally, Chapter 16 brings us to the calculus of **vector fields** in $\mathbb{R}^2$ and $\mathbb{R}^3$, which might just be some of the most widely-applied material in the course. We will begin by introducing vector fields before moving on to several methods for integrating them, and discussing what such integrals might be used to compute.

1.2 Prerequisites

In reading the previous section, you might've noticed that much of what we're going to do in the course is differentiate and integrate. If you feel comfortable with the material from your previous calculus courses, this is very good news. It turns out that, with only very small changes, most of the calculus that you already know applies directly to this course. If you're not so comfortable with that material, you should spend some time reviewing and solidifying that knowledge as we proceed through the course, as we will often call upon it. In particular, here are some of the topics that will come up:

- Limits
  - Evaluating limits by "plugging in"
  - Evaluating limits by simplifying first
  - Evaluating limits by conjugation
  - Evaluating limits using the squeeze theorem
  - Evaluating limits at infinity
  - Evaluating limits with L'Hopital's rule
- Derivatives
  - Sum, product, and quotient rules
  - The chain rule
  - Implicit Differentiation
  - Derivatives of trigonometric and inverse trigonometric functions
  - Derivatives of exponential functions
  - Derivatives of logarithmic functions
- Integration
  - Integration by substitution
  - Integration by parts
  - Integrals of products of sines and cosines

Additionally, you should have already covered the following topics from Chapter 12 in your previous calculus courses:

- 12.1
  - Plotting points in $\mathbb{R}^3$
  - Computing the distance between two points in $\mathbb{R}^3$
  - Equations of spheres in $\mathbb{R}^3$
- 12.2
  - Plotting vectors in $\mathbb{R}^2$ and $\mathbb{R}^3$, given their components
  - Adding and subtracting vectors, both algebraically and graphically
  - Computing the magnitude (or length) of a vector
  - Multiplying a vector by a scalar, both algebraically and graphically
  - Representing a vector as either a set of components in angle brackets; or as the sum of multiples of the vectors $\vec{i}$, $\vec{j}$, and $\vec{k}$
  - Scaling a vector so that its magnitude is 1, i.e., so that it becomes a unit vector
- 12.3
  - Computing the dot product of two vectors in $\mathbb{R}^2$ or $\mathbb{R}^3$ using their components
  - Computing the dot product of two vectors given their lengths and the angle between them
    - In particular, using the dot product to compute the angle between two vectors
- 12.4
  - Computing the cross-product of two vectors in $\mathbb{R}^3$, and interpreting this cross-product as a vector orthogonal to the original pair

2 Course Mechanics

2.1 How to Study

In many ways, studying for a class is a thoroughly personal act. That is, you've spent many years studying for math courses, and in that time you've probably learned a lot about how you learn best. However, I am still regularly asked about how to best prepare for assignments in my course, so in this section I've compiled a list of suggestions, many of which are backed by education research. As ever, though, these are suggestions—please feel free to take or leave them as you see fit.

2.1.1 Spread Out Your Study Time

Perhaps the most important piece of research-backed advice I can give you is that spreading out study time is more effective than holding long study sessions. For example, suppose that you plan to study for this course for about five hours per week. It would be better to spend, say, an hour each day working on the material than studying for 2.5 hours twice a week, or, even worse still, studying for five hours straight in one day.

There are a few reasons for this. When you learn math, you are continually building new skills. But skill building takes time to sink in; it doesn't come all at once. A surprising thing about learning is how much of it occurs in the background, when you're not actively studying something. So, after you learn a new skill, it's good to just let that information sit for a little while. It's good to shift gears, and let your brain integrate that knowledge, before you move on to the next task. If you start to get a handle on one skill and then jump right to the next one, you may find that when you return to the first your footing has gotten a little less firm. If you keep pushing, you may find yourself sort of tumbling down the rabbit hole, getting lost and buried under all there is to do. Let your brain breathe, and take small steps to get where you're going. Be as gentle with yourself as you can be. Breaking up your studying will help in this regard.

2.1.2 Work in Intervals—and Take Breaks

Research (and the experience of people who have learned to perform various skills at high levels) suggests that the most effective way to practice a skill is in several relatively brief, targeted sessions throughout the day that focus on a specific task, and that are just at the edges of your ability in that task. Once your focus just begins to slip, it's time to take a break, and come back later.

Seriously: don't be afraid to take breaks when it's time. You can't work well without also resting well. When you take a break, take it completely. Get up from your desk, stretch, move around, go somewhere else, and clear your head. Do something mindless that'll carry you away from your work. On average, five minutes is all it takes for people to feel rested enough to get back to work, but that five minutes of space is *crucial*. Be kind to yourself, and kind to your very busy brain.

2.1.3 Keep a Reserved Study Space and/or Ritual

Another strong recommendation backed by research is to, if possible, find a space that you utilise only when you're studying. If you have such a space, entering it will put you in the right frame of mind to practice and work on your skills; and exiting it will help you to detach from your work when you've finished a study session. However, if you cannot find such a space, or if one is not available to you, it is good to have something in your multipurpose study space that signals it's time to study. For example, at home I have a small lamp on my desk—a desk that I use for many tasks, from watching movies, to paying bills, to grading, to studying—which gets turned on *only* when I'm studying. If the lamp is on, I'm working; and when I've finished, I turn it off. This small act transforms my workspace into a state that signals to my brain that I plan—or don't plan, as the case may be—to be in a working headspace.

2.1.4 Study With An Eye Toward Concepts

You're going to learn a lot of facts throughout this course, and trying to remember them all as isolated facts through rote memorization is a tall order. Research suggests that a better way to remember those facts is through a conceptual understanding of the material. Here's an example to illustrate the idea:

You may recall the the derivative $f'(a)$ of a single-variable function $f(x)$ at the value $x = a$ is, by definition:

$$f'(a) = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h} = \lim_{x\to a} \frac{f(x) - f(a)}{x - a}$$

Without any conceptual context, this can be a challenging fact to memorize, especially given that there are two equivalent ways to define $f'(a)$ (not to mention the other mountains of facts one is also trying simultaneously to memorise in a calculus course). One risks conflating the two definitions; forgetting the order of the terms in the numerator of the difference quotient; forgetting what each denominator should be; etc.

But a conceptual understanding of the derivative is what cements this definition in our minds. If one recalls that $f'(a)$ is defined as it is to yield the slope of the line that lies tangent to the graph of $f(x)$ at the point $(a, f(a))$, then the definition is much easier to remember: the difference quotient that lies under each limit represents the slope of a secant line passing through the points $(a, f(a))$, and $(a+h, f(a+h))$ or $(x, f(x))$, respectively, on the graph of $f(x)$. As we slide the second point closer to $(a, f(a))$, our secant line approaches a tangent line, which explains the limit in each case. A brief sketch of this situation in the corner of some scrap paper is enough to remind one of the definitions above; it's the concept which produces the fact.

Reading a fact and remembering it for a few minutes is quick and easy. But putting in some time to understand *why* a given fact is true often makes longer-term recall much easier. So, when possible, study with an eye toward such conceptual understanding.

To really drive things home, I can't resist another popular example. Have a friend read out the following list of letters to you, and see how many you can memorise:

L - I - E - R - D - N - F - E - A - T - I - F

How many did you get? Research suggests something like five to seven letters is probably about what you got. Now try again, with the following list of letters:

D - I - F - F - E - R - E - N - T - I - A - L

Did you have better luck? The two lists of letters are the same (compare to see this), but having a concept attached to the second version of the list—the word "differential"—made memorising this list *much* easier. Even if you were able to memorise the first list of letters in its entirety, the second list would make it much easier to remember that same list for days, weeks, and even months.

2.1.5 Work on Novel Problems—and Start The Same Day

Another broad recommendation I can give is to actively engage with the material as you are studying it. This can mean lots of different things. For starters, research suggests that it's a good idea to do some homework on lecture material the same day that you hear it in class. Indeed, if you decide to wait a couple days before returning to lecture material, you'll probably find that some of your understanding (maybe even much of it) has ebbed away in the meantime, even if the material made a lot of sense during class. Engaging with it sooner will help you keep the progress that you made during class.

Engaging with the material also means doing problems, more generally. If I'm doing my job the way I would like, I truly hope that by the end of each of our classes you'll be able to walk away feeling like the material makes sense. I hope that you'll feel confident in your understanding of the concepts of the course, and that you'll feel that the in-class examples are more-or-less approachable. But also, it's important to note that this confident feeling can be a trap. It isn't until you sit down and try to solve some problems on your own that you really know how well you understand the material. Or, to put it another way, true understanding begins when you apply what you've learned. It's a good idea to read through example problems a couple of times to get an idea of how similar problems might be solved, but you need to head to the edge of your abilities to make real progress and gain real understanding—you need to start solving novel problems. Fortunately, our book is full of them.

2.1.6 Reflect as You Study

It might just sound like the previous subsection is an elaborate way of saying "do your homework." But that's only part of the message. It's good to be able to do the homework, but just doing the problems is sort of a bare minimum. There's more to engaging with the material than that:

- As you're doing the homework, keep mental notes about the different types of problems you encounter. Are you struggling with particular types? Try and do some more of those.

- Don't be afraid to try problems that *aren't* on the homework. The homework is only a bare minimum guide for the types of problems you should know how to solve for exams; it may not give you complete understanding of a topic on its own, nor is it meant to. Do as many or as few problems as you need to feel comfortable with a given topic.

- Ask yourself the different ways that a given problem might vary. Can you solve those variations?

- Look for novel problems to solve. It's good to be able to solve the example and homework problems, but once you know how to solve them, the only way to keep improving your skills is to branch out. That is: after you understand how they're done, rehearsing solutions to solved problems doesn't build your skills the way that solving new ones does.

- When you first start working through problems in a given section, it's a good idea to use your notes as a guide. As you feel your skills improving, see how much you've internalised the material by doing some without your notes, too. Eventually, you shouldn't need your notes.

- How do the different sections interact? Does the material you learned in section X apply to section Y? How do these sections square with one another?

- Keep track of the sections and problem types that you find easier or more difficult, and return to the latter to improve your skills, even as the course moves on.

- It's also okay to move on for a while once you *do* understand how to do particular types of problems. It's better to push your understanding outward and learn new things than to rehearse things that are already familiar and comfortable.

- That said, it's a good idea to glance back at all of the types of problems you've learned to solve from time to time, to keep your skills from getting rusty.

This may sound like a great deal of work, but a lot of this boils down to: ask yourself what types of problems you know or don't know how to solve, and try to improve your skills on the latter. The above are suggestions for how to do so, and how much or how little you do of each depends greatly on your wants and needs.

2.1.7 Learn From Your Peers

Finally, I want to say a little about study groups and asking for help. Finding people to study with can be a wonderful help. Talking with others and taking time to practice explaining key concepts and problems can really help to both cement knowledge that you've already gained, and to point out flaws in your understanding. So, I would absolutely encourage you to find others to study with, if you're so inclined. Just make sure that the people you find to work with are people that you can both communicate well with about math, and are people who really do want to do work when you get together.

Study groups are also a great place to start when you need help. I want to be very clear here: anytime you want to come by my office hours, talk to me before or after class, or email me about questions you may have, I'm more than happy to talk with you about course material, individual problems, prerequisite material, or just general questions about math. Class time is always limited and compressed because of how much there is to do throughout the semester, but I'm always happy to take some time with you individually to expand on anything you might have questions about.

All of that said, I think your peers are probably the best place to start with your questions. Indeed, it has been more than a decade since I first learned the material in this course, so I have long since forgotten what it was like to be seeing this content for the first time. That is, I've forgotten much of what it was that I used to not know. So, sometimes it can be tricky for me to communicate what you need to hear in exactly the ways that you need to hear it. Your peers are probably going to have questions and struggles that are very similar to yours, and thus they may be in the best position to talk to you and answer your questions in exactly the ways you may need to hear.

2.1.8 Formulate Questions as Explicitly As Possible

When you do need help with a particular question, you should strive to be clear and explicit about what it is you don't understand, or what you're struggling with. Sometimes, just formulating the question well can help point you toward an answer. If your question is about a particular problem, it's also important to explain what you've tried to do so far, and to explain how far toward a solution you've gotten.

2.1.9 Stuck? Move On For a While

Don't let yourself get too hung up on any one problem that you might encounter. If you find yourself staring at a page for twenty minutes with no idea of where to go on a particular problem, just move on for the time being. An idea may come to you later, after you've solved similar problems; or maybe even later in the course, after you've gained more experience. You can also always ask your peers or me about it, too! I'm always happy to talk math!

3 Miscellany

3.1 Calculating Your Current Grade

You may wish to track your grade throughout the course. I will use an example to illustrate how this may be done. Recall from the syllabus that:

Course Component	% of Grade
Office Hours Visit	5%
Class Participation	10%
Quizzes	20%
Exam I	15%
Exam II	15%
Exam III	15%
Final Exam	20%

Suppose that you've just recieved your first exam back, with a grade of 88/100. Suppose also that you have an overall average of 5.3/6 on your quizzes, which you've been told will curve to a 95/100. You made your office hours visit, and you've been participating in class regularly, so you can expect that your score for those portions of your grade are perfect. With all of this information in mind, you may compute your current grade in the course as follows:

$$\left[\frac{\left(\frac{100}{100}\right)(5)+\left(\frac{100}{100}\right)(10)+\left(\frac{95}{100}\right)(20)+\left(\frac{88}{100}\right)(15)}{50}\right]\cdot 100 = 94\%$$

Let's break this calculation down. Think of your final grade as a maximum of 100 points in the course, and think of each of the percentages in the table above as a maximum number of points toward that hundred that may be earned in each category. In other words, your final grade consists of up to 5 points for an office-hours visit; up to 10 points for participating in class discussion; up to 20 points for your quiz average; up to 15 points for each of the regular semester exams; and up to 20 points for your final exam grade.

Now, the calculation above says that, so far, you have earned 100% of the five available office hour visit points, 100% of the ten available class participation points, 95% of those twenty available quiz points, and 88% of the fifteen points available from the first exam. We add these to get the total number of points that you have earned so far. On the other hand, the maximum number of points you *could* have earned up to this point in the course is 50, so to get your grade as a percentage, we divide the number of points you've earned so far by 50 and multiply by 100. Thus, your current grade is a 94%.

Exercise. What will your grade be if you get a 63 on the second exam and your quiz average remains the same? (Solution: 87.15%)

3.2 Extra Credit

No extra credit will be offered.