Putnam Seminar 2004

We continue with some problems from "Mathematical Miniatures" by S. Savchev and T. Andreescu. You can find the problems we looked at in the Putnam Seminar last year by going to **http://www.math.ou.edu/~jalbert/putnam/putnam.html** .

Helly's Theorem for One Dimension

1. During a certain lecture, each of the students in the room fell asleep exactly once. For each pair of students, there was some moment when both were sleeping simultaneously. Prove that, at some moment, all the students were sleeping simultaneously.

The preceding problem is essentially equivalent to the following (which is Helly's Theorem on the line):

2. If each pair of a finite collection of intervals on a line has a common point, then all these intervals have a common point.

Note: the two-dimensional version of Helly's theorem says that if we have a finite collection of convex figures in the plane, any three of which have a common point, then all the figures must have a common point.

3. Finitely many convex figures are given in the plane, and each pair of them has a common point. Prove that for each line $\mathcal{L}$ in the plane there exists a line parallel to $\mathcal{L}$ that intersects all these figures.

4. Finitely many arcs are given on a circle. Each pair of them has a common point. Prove there exists a line through the center of the circle that intersects all these arcs.

5. Finitely many convex figures are given in the plane, and each pair of them has a common point. Prove that for each point O in the plane there exists a line through O that intersects all these figures.