**Some problems from the** *American Mathematical Monthly*

**1.** The perimeter of a triangle $ABC$ is divided into three equal parts by three points $P$, $Q$, $R$. Show that

$$\text{Area}(\Delta PQR) > \frac{2}{9}\,\text{Area}(\Delta ABC)$$

and that the constant 2/9 is the best possible.

**2.** Find all pairs of positive integers $m$, $n$ such that $\phi(m)|n$ and $\phi(n)|m$, where $\phi$ denotes Euler's function.

**3.** Let $S$ be the boundary of the unit square $[0,1] \times [0,1]$ in $\mathbf{R}^2$. Suppose $f$ is a continuous real-valued function on $S$ such that $f(x,0)$ and $f(x,1)$ are polynomial functions of $x$ on $[0,1]$ and such that $f(0,y)$ and $f(1,y)$ are polynomial functions of $y$ on $[0,1]$. Prove that $f$ is the restriction to $S$ of a polynomial function of $x$ and $y$.

**4.** Suppose $n$ points are independently chosen at random on the perimeter of a circle. What is the probability that all the points lie in some semicircle?

**5.** A population consisting of particles of various types evolves in time according to the following rule: Each particle is deemed to belong to a unique generation $n = 1, 2, 3, \ldots$. Each particle produces a certain number of "offspring" particles, and, for each $n$, generation $n+1$ comprises the totality of offspring of the particles in generation $n$. A particle of type $i = 0, 1, 2, \ldots$ produces exactly $i+2$ offspring, one each of types $0, 1, 2, \ldots, i+1$. Let $N(n,k)$ denote the number of particles in the $n$th generation when the first generation consists of a single particle of type $k$. Find a formula for $N(n,k)$.