1. (This is a Putnam problem, but I’m not sure from what year:)
   Consider the sequence $u_n$ defined by $u_0 = u_1 = u_2 = 1$, and
   
   $$\det \begin{pmatrix} u_{n+3} & u_{n+2} \\
   u_{n+1} & u_n \end{pmatrix} = n!,$$

   for $n \geq 0$. Prove that $u_n$ is an integer for all $n$.

2. (Not actually a Putnam problem:)
   Define
   
   $$x_n = \sqrt{1 + \sqrt{1 + \sqrt{1 + \cdots + \sqrt{1}}}},$$

   where there are $n$ 1’s on the right-hand side. Show that the sequence $x_n$ converges to a limit, and find the limit.