

Putnam Seminar — week 1 — Sept. 16, 2015

(A1, 2015) Prove that every nonzero coefficient of the Taylor series of

$$(1 - x + x^2)e^x$$

about $x = 0$ is a rational number whose numerator (in lowest terms) is either 1 or a prime number.

(A3, 2015) Let $a_0 = 5/2$ and $a_k = a_{k-1}^2 - 2$ for $k \geq 1$. Compute

$$\prod_{k=0}^{\infty} \left(1 - \frac{1}{a_k}\right)$$

in closed form.

(B4, 1995) Evaluate

$$\sqrt[8]{2207 - \frac{1}{2207 - \frac{1}{2207 - \dots}}}$$

Express your answer in the form $\frac{a+b\sqrt{c}}{d}$, where a, b, c, d are integers.