Note: I've translated the following from the French original (with a few Latin footnotes). I thought it was an interesting snapshot of how the differential calculus was viewed to those who lived through the days of its discovery.

Preface to the textbook

"Analysis of the infinitely small, for the understanding of curved lines"

(published anonymously by the Marquis de L'Hopital in 1696)

The Analysis that we explain in this work makes use of common analysis, but differs from it greatly. Ordinary Analysis only deals with finite quantities: ours penetrates all the way to infinity itself. It compares infinitely small differences to finite quantities; it discovers the relations between these differences: and in so doing it makes known the relations between finite quantities, which when compared with the infinitely small are themselves like infinities. One could even say that this Analysis extends beyond infinity: for it does not stop at infinitely small differences, but discovers the relations between differences of these differences, and further those between third differences, fourth differences, and so on, without ever finding a termination that could halt it. So that it does not only embrace the infinite, but the infinite of the infinite, or an infinity of infinities.

Only an Analysis of this nature could lead us to the true principles of curved lines. For, as curves are nothing but polygons with an infinity of sides, and are only distinguished from each other by the difference of the angles that these infinitely small sides form with each other; only the Analysis of the infinitely small can determine the position of these sides and so obtain the curvature which they form, which is to say the tangents of these curves, their normal lines, their points of inflection and cusps, the rays which are reflected by them, and those which are refracted by them, etc.

The polygons inscribed in or circumscribed on curves, which through the infinite multiplication of their sides, finally become one with them, have always in the past been taken to be the same as the curves themselves. But that was as far as people had gone: it is only since the discovery of the Analysis which we treat here, that we have really felt the extent and the fertility of this idea.

What we have from the Ancients on these matters, principally from *Archimedes*, is assuredly worthy of admiration. But besides the fact that they only touched on very few curves, and on those only very lightly; almost all their propositions are special cases and without order, which does not allow us to perceive any regular and step-by-step method. We can not, however, legitimately reproach them for this: they had need of an extreme force of genius* to pierce through so many obscurities, and to be the first to explore entirely unknown lands. If they did not go far, if they traveled by roundabout routes, at least (despite what *Vieta* says†) they never went wrong: and the more difficult and prickly the paths which they followed, the more admirable it was of them not to get lost on them. In a word, it does not seem that the Ancients could have done more in their times; they did what the best minds of today would have done in their place; and if they were in our place, one must believe that they would have the same views that we do. All this is a consequence of the natural equality of minds among men and of the necessary succession of discoveries.

So it is not surprising that the Ancients did not go farther; but one cannot be astonished enough at the way in which great men, even men no doubt as great as the Ancients, have remained there so long; and through an almost superstitious admiration for their works, have been content to read and comment

* *Although I have read Archimedes' treatise on spirals a number of times, and have exerted all the powers of my mind to understand the the art of the extremely subtle proofs of tangent spirals; nonetheless I frankly confess that I have never returned from my contemplations of it without the scruple still remaining in my mind that I have not grasped the total force of these demonstrations, etc.*, from Bullialdus' preface to *De Lineis Spiralibus.*

† *If Archimedes has concluded correctly, then Euclid has concluded wrongly, etc.*, from his *Supplementum Geometriae.*

on them, without allowing themselves any other use of their lights, than what was needed to follow them; without daring to commit the crime of sometimes thinking for themselves, and carrying their view beyond what the Ancients had discovered. So it was that many people worked and wrote, and books proliferated, and yet nothing advanced: all the work of several centuries served only to fill the world with respectful commentaries and repeated translations of originals that were often of rather little value.

Such was the state of Mathematics, and above all of Philosophy, until M. *Descartes*. This great man, impelled by his genius and by the superiority which he felt, took leave of the ancients so as to only follow reason, the same reason that the Ancients had followed; and this fortunate boldness, which at the time was called revolt, has given us an infinity of new and useful views on Physics and Geometry. Thus our eyes were opened, and we advised ourselves to think.

Speaking only of Mathematics, which is our subject here, M. *Descartes* began where the Ancients had left off, and he started by solving a problem on which *Pappus* says* they had all been stuck. We know how far he carried Analysis and Geometry, and how much the alliance he made between them facilitated the solution of an infinity of Problems which had seemed impenetrable before him. But since he applied himself mainly to the solving of equations, he did not pay attention to curves except as they could be useful to him for finding roots: so that, ordinary Analysis being sufficient to him for that, he did not think at all of looking for any other. He did not however neglect to make good use of it in his research on Tangents; and the method he discovered for that seemed to him so beautiful that he found no difficulty in saying† that *this Problem was the most useful and the most general, not only that he knew, but even that he had any desire to ever know in Geometry.*

Since the Geometry of M. *Descartes* had made the construction of Problems by solving equalities very fashionable, and since he had done great work on that; most Geometers applied themselves to it, and also made new discoveries, which to this day continue to grow in number and perfection.

As for M. *Paschal*, he took a completely different viewpoint: he examined curves in and of themselves, and in the form of polygons; he researched the lengths of some of them, the areas that they enclosed, the volume that these areas described, the centers of gravity of all of these, etc. And by the sole consideration of their elements, that is to say of the infinitely small, he discovered general Methods, so much the more surprising in that he seemed to have arrived at them by brainpower and not analysis.

A short time after the publication of the Method of M. *Descartes* for tangents, M. *de Fermat* also found one, which M. *Descartes* himself finally admitted‡ to be simpler in many situations than his own. It is nevertheless true that it still was not as simple as M. *Barrow* afterwards made it by considering more closely the nature of polygons, which presents naturally to the mind a little triangle made of a small piece of the curve, comprised between two infinitely close ordinates, the difference of these two ordinates, and the difference of the corresponding abscissas; and this triangle is similar to that formed by the tangent, the subtangent, and the ordinate: so that by a simple Analogy this method spares us from all the calculations required by that of M. *Descartes*, and which even [Fermat's] method had previously required.

M. *Barrow*§ did not stop there, he invented also a kind of calculus suited to this method; but it was necessary for him, as in the method of M. *Descartes*, to remove fractions and make all radical signs disappear before using it.

The shortcomings of this method were overcome by that introduced by the celebrated M. *Liebniz*‖; and this learned Geometer has started where M. *Barrow* and the others had left off. His calculus led him

* at the beginning of Book 7 of his Mathematical *Synagoge* or *Collection*.

† in Book 2 of his *Geometry*.

‡ in his letter to M. de Beaune, Feb. 20, 1639; letter no. 71, pp. 409–416 in volume 3 of Clerselier's edition of his letters.

§ in his *Lectiones Geometricae* (London, 1670), p. 80.

‖ *Acta Eruditorum Leipzig* (1684), p. 467.

into hitherto unknown territories; and there he has made discoveries which astonish the most accomplished Mathematicians of Europe. M^{rs}. *Bernoulli* have been the first to perceive the beauty of this calculus: they have carried it to a point which has put them in a position to surmount difficulties that one would never have dared to attack before.

The reach of this calculus is immense: it applies to mechanical Curves as well as to geometric ones; it is not troubled by radical signs and often even finds them convenient; it can be extended to as many indeterminates as one pleases; and the comparison of infinitely small quantities of all types is equally easy for it. And from it is born an infinity of surprising discoveries about Tangents both curved and straight, about questions of maxima and minima, about points of inflection and cusp points of curves, about Evolutes, about Caustics by reflection and by refraction, etc. as we will see in this work.

I divide it into ten Sections. The first contains the principles of the Calculus of differences. The second explains how to use this [Calculus] to find tangents to all sorts of curves, no matter how many indeterminates there are in the equation which expresses them, even though M. *Craig** had not believed that it could be extended to mechanical or transcendental curves. In the third, how it is used to resolve all questions about maxima and minima. In the fourth, how it gives points of inflections and cusps of curves. In the fifth is revealed how to use it to find the Evolutes of M. *Huygens*, for all sorts of curves. The sixth and seventh explain how it gives Caustics, by reflection as well as refraction, of which the illustrious M. *Tschirnhaus* is the inventor, also for all sorts of curves. The eighth explains how to use it to find the the points on curves which touch a given family of infinitely many other curves or straight lines. The ninth contains the solution of several Problems which depend on the preceding discoveries. And the tenth consists of a new way of using the Calculus of differences on geometric curves: from which one deduces the methods of M^{rs}. *Descartes* and *Hudde*, which is only suited to these types of curves.

It should be noted that in Sections 2, 3, 4, 5, 6, 7, 8, there are only a very few Propositions; but they are all general ones, and like other Methods it is easy to apply it to as many particular propositions as you wish: I use it only on certain choice examples, persuaded that in Mathematics it is only methods that are profitable, and that books which only consist of details or particular propositions, are good only for wasting the time of those who write them, and those who read them. Therefore I have only added the Problems in the ninth section because they are sufficiently interesting and very universal. In the tenth section there are again only the Methods of the Calculus of differences given in the manner of M^{rs}. *Descartes* and *Hudde*; and if they are quite limited, one sees from all that precedes them that this is not a defect of this calculus, but rather of the Cartesian Method to which it is subjected. On the contrary nothing proves the immense usefulness of this calculus better than all this variety of methods, and for the small amount of work it requires, we will see that it recovers all that can be recovered of the Method of M^{rs}. *Descartes* and *Hudde*, and that the universal justification that it gives of the use that one makes there of arithmetic progressions, leaves nothing to be desired as to the infallibility of this Method.

I had intended to add also a Section to make known the marvelous use of this calculus in Physics, to what point of precision it can be carried, and how much usefulness can be drawn from it by Mechanics. But an illness prevented me: however it will not be lost to the public, who will have some day both the principal and interest.

In all this there is yet only the first part of the calculus of M. *Liebniz*, which consists in descending from whole quantities to their infinitely small differences, and in comparing these infinitely small quantities, of whatever type they may be, among themselves: this is what is called *differential Calculus.* For the other part, which is called *integral Calculus*, and which consists of ascending from these infinitely small quantities to the wholes of which they form the differences, that is to say of finding their sums, I had also intended to describe it. But M. *Liebniz* having written me that he was working on a Treatise which he titles *De Scientia Infiniti*, I have not wanted to deprive the public of so fine a Work which should include all that is most surprising about the inverse Method of Tangents, about Rectifications of curves, about the Quadrature of

* in his *De figurarum quadraturis*, part 2.

the areas they enclose, and that of the surfaces of the bodies they describe, about the dimensions of these bodies, about finding centers of gravity, etc. I only even make this public now because he has asked me to in his letters, and because I believe it necessary to prepare readers' minds for comprehending all that they will be able to discover in the future on these matters.

For the rest I acknowledge owing much to the lights of the M^{rs}. *Bernoulli*, especially those of the young one who is presently Professor at Groningue. I have freely used their discoveries and those of M. *Liebniz* . For this reason I consent to their claiming any part of it that they wish, contenting myself with whatever they see fit to leave to me.

There is also justice owed to the learned M. *Newton*, and which M. *Liebniz* has himself rendered* to him: That he had also found something similar to the differential Calculus, as it appeared in the excellent Book titled *Philosophia naturalis principia Mathematica*, which he gave us in 1687. which is almost all of this calculus. But the Notation of M. *Liebniz* makes his method much easier to use and more expeditious; besides being a marvelous aid in many situations.

As the final page of this Treatise was being printed, the Book of M. *Nieuwentiit* fell into my hands. Its title, *Analysis infinitorum*, made me curious enough to read through it: but I found that it was much different than this one; for besides the fact that that author makes no use of the Notation of M. *Liebniz*, he absolutely rejects second differences, third differences, etc. As I have built the best part of this Work on that foundation, I would have felt obliged to respond to his objections, and to explain how they are not very solid, if M. *Liebniz* had not already fully accomplished that in the Leipsic Acts†. Moreover the two hypotheses or suppositions that I make at the beginning of this Treatise, and on which alone it relies, seem to me so evident, that I do not believe that they could leave any doubt in the minds of attentive Readers. I could even have easily proved them after the manner of the Ancients, if I had not decided to say little about things which are already known, and to address myself principally to those which are new.

* in "Considerations sur la différence qu'il y a entre l'analyse ordinaire et le nouveau calcul des transcendantes", *Journal des Savants*, Aug. 30, 1694.

† *Acta Eruditorum* (1695), p. 310 and 395.