MATH 3333
Worksheet 3

1. Suppose that $\mathbf{u}$ and $\mathbf{v}$ are vectors in R^n and A is an $n \times n$ matrix. Suppose that $A\mathbf{u}$ and $\mathbf{v}$ are orthogonal with respect to the standard inner product on R^n. Show that $\mathbf{u}$ and $A^T\mathbf{v}$ are orthogonal. (Hint: remember that if $\mathbf{p}$ and $\mathbf{q}$ are vectors in R^n, then the standard inner product of $\mathbf{p}$ and $\mathbf{q}$ is defined to be $\mathbf{p}^T\mathbf{q}$.)

2. Suppose that $\mathbf{u}$ and $\mathbf{v}$ are vectors in R^n and A is an $n \times n$ matrix. Suppose that $A\mathbf{u} = \mathbf{0}$ and suppose that $\mathbf{v} = A^T\mathbf{w}$ for some vector $\mathbf{w}$ in R^n. Show that $\mathbf{u}$ and $\mathbf{v}$ are orthogonal with respect to the standard inner product on R^n.

3. Suppose $L : V \to W$ is a linear transformation and $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ are vectors in V. Suppose $\{L(\mathbf{v}_1), L(\mathbf{v}_2), L(\mathbf{v}_3)\}$ is linearly independent. Show that $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is linearly independent.