MATH 3333
Worksheet 2

1. Suppose that $S = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is a linearly independent set of vectors in a vector space V. Let $\mathbf{w}_1 = \mathbf{v}_1 + \mathbf{v}_2$, $\mathbf{w}_2 = \mathbf{v}_1 + \mathbf{v}_3$, and $\mathbf{w}_3 = \mathbf{v}_2 + \mathbf{v}_3$. Show that the set $T = \{\mathbf{w}_1, \mathbf{w}_2, \mathbf{w}_3\}$ is also linearly independent.

2. Let M_{23} be the vector space of all 2×3 matrices. Find a basis for M_{23} (with proof), and give the dimension of M_{23}.

3. Suppose that $S = \{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n\}$ is a linearly independent set of vectors in R^n. Suppose that A is a non-singular $n \times n$ matrix, and let $\mathbf{w}_1 = A\mathbf{v}_1$, $\mathbf{w}_2 = A\mathbf{v}_2$, $\ldots$, $\mathbf{w}_n = A\mathbf{v}_n$. Show that $T = \{\mathbf{w}_1, \mathbf{w}_2, \ldots, \mathbf{w}_n\}$ is linearly independent.

4. Suppose that $S = \{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n\}$ is a set of vectors that spans R^n. Suppose that A is a non-singular $n \times n$ matrix, and let $\mathbf{w}_1 = A\mathbf{v}_1$, $\mathbf{w}_2 = A\mathbf{v}_2$, $\ldots$, $\mathbf{w}_n = A\mathbf{v}_n$. Show that $T = \{\mathbf{w}_1, \mathbf{w}_2, \ldots, \mathbf{w}_n\}$ spans R^n.