

**MATH 3333**  
**Worksheet 1**

1. Suppose  $A$  is an  $m \times n$  matrix, and  $AA^T = O$ , where  $O$  is the  $m \times m$  matrix whose entries are all zero. Prove that all the entries of  $A$  are zero.
  
2. (a) Suppose  $A$  is an  $m \times n$  matrix with a row consisting entirely of zeroes, and  $B$  is any matrix with  $n$  rows. Prove that  $AB$  has a row consisting entirely of zeroes.  
(b) Suppose  $A$  is an  $m \times n$  matrix with a column consisting entirely of zeroes, and  $B$  is any matrix with  $m$  columns. Prove that  $BA$  has a column consisting entirely of zeroes.
  
3. A matrix is defined to be *symmetric* if it is equal to its own transpose. Suppose  $A$  and  $B$  are symmetric matrices, and suppose that  $AB = BA$ . Prove that  $AB$  is symmetric.
  
4. Suppose  $A$  is an  $m \times n$  matrix. Prove that
  - (i)  $AA^T$  and  $A^T A$  are symmetric, and
  - (ii)  $A + A^T$  is symmetric.