Review for Final Exam

The final exam is comprehensive, but will be weighted more towards the material in the latter half of the course (i.e., from Chapter 15 onwards). You can refer to the review sheets for the second and third exams for an outline of what will be covered from chapters 14, 15, and 16, and sections 17.1 through 17.4. I will probably not ask any questions specifically on chapter 13, but if you take a minute to go back and look at chapter 13, you will see that the topics covered there are crucial for the rest of the course. I recommend going back and reading through this chapter at least once, doing sample problems when you feel it would help. You don't want to mess up a problem on Stokes' theorem because you got the dot product mixed up with the cross product, or the equation of a plane mixed up with the equations of a line, or r in cylindrical coordinates mixed up with ρ in spherical coordinates. (I was guilty of the last mistake in working the last example in class on Dec. 7, by the way.)

Here is a review guide for sections 17.5 through 17.9.

17.5. Curl and divergence. We covered the entire section. The symbolic expressions in equations 2 and 10 are helpful for remembering the definitions of curl and divergence. Remember, the curl of a vector field is a vector field, and the divergence of a vector field is a scalar field (i.e., a function of x, y and z whose values are numbers, not vectors). Also remember that the operation of taking a gradient is different from either the divergence or curl operation. The gradient of a function is a vector field. In particular, if $f(x, y, z)$ is a function whose values are numbers, it would make sense to talk about the divergence of ∇f, but it would not make sense to talk about the divergence of f itself.

17.6. Parametric surfaces and their areas. We covered the entire section, although I didn't actually get to the material in the subsection on "Surface area of a graph of a function" until the next-to-last day of class. The key idea here is that when you want to find a parametric representation for a surface, you should look for a natural way to specify the location of a point of the surface by giving two numbers (usually called u and v). On a sphere, for example, a natural way to specify the location of a point would be by giving its polar coordinates ϕ and θ (these correspond roughly to the latitude and longitude of a point on the earth). The procedure is a natural generalization of finding a parametric representation of a curve, where you look for a natural way of specifying the location of a point on the curve by giving one number (usually called t).

17.7. Surface integrals. We covered almost the entire section in class. You can skip the material on the last two pages about electric flux and heat flow, although if you've already learned about those topics in other courses it would be helpful to read about them here.

Remember, just as there are two kinds of line integral, there are two kinds of surface integral: the surface integral of a function, defined in equation 1, and the surface integral of a vector field, defined in equation 7. It is the latter kind that appears in Stokes' theorem and the divergence theorem. See Quiz 7, where I made a point of emphasizing the difference between these two kinds of surface integral.

When computing a surface integral you would not use its definition, but would do an integral in parameter space, using either equation 3 for the surface integral of a function or equation 8 or 9 for the surface integral of a vector field. Actually, there was yet another way of computing surface integrals which I went over in class: if a surface S is given by the equation $f(x,y,z) = 0$ and lies above the region R in the xy-plane, then

$$\int\int_S g(x,y,z)\ dS = \int\int_R g(x,y,z)\frac{|\nabla f|}{\left|\frac{\partial f}{\partial z}\right|}\ dA$$

and

$$\int\int_S \mathbf{F}\cdot d\mathbf{S} = \int\int_R \mathbf{F}\cdot\nabla f\frac{1}{\frac{\partial f}{\partial z}}\ dA.$$

Sometimes one of these formulas is more convenient to use than the others, but you can always get away with using the formulas in either equation 3 or equation 9, so those are the only two it's absolutely necessary to learn.

17.8. Stokes' theorem. You can skip the proof of Stokes' theorem on p. 1158 and the second and third paragraphs on p. 1160. Otherwise you should read the whole section.

17.9. Divergence theorem. You can skip the proof of the divergence theorem and all the material which comes after Example 2.

On the other hand, let me put in a good word here for proofs. Reading them is a time-consuming activity, and the meaning of a proof doesn't always sink in on first reading (or second or third), even after you are satisfied you understand all the steps of the proof. On the other hand, there's no alternative to reading the proof of a formula if you really want to grasp its meaning. And even after a first reading of a proof, if nothing else, you've internalized the formula being proved to the extent that it will be harder to make stupid mistakes using it. So while you can safely skip the proofs if you're short on study time (and who isn't?), reading them does pay off, in ways you might not even be aware of.