Review for Final Exam

The final will be a little less than twice as long as the in-class exams. As on the in-class exams, the ratio of "computational" problems to "proof" problems will be about 90 percent to 10 percent. It's actually not always easy to distinguish between the two types of problems, but you can get a good idea of what I mean by looking at the previous exams.

Below I've listed the main ideas of the course, with references to section numbers in the text in parentheses. Think about them about a bit, trying to keep the overall structure of the course in mind, while going over the problems on the homework, quizzes, and tests.

- Putting a system of equations in matrix form (1.1).
- Properties of matrix operations: addition of matrices, multiplying a matrix by a scalar, multiplication of matrices, transposes of matrices (1.2, 1.3, 1.4)
- Nonsingular matrices, inverses of matrices. (1.5, 2.2, 3.8, 5.2. See also blue boxes on p. 211 and p. 311)
- Definition and basic properties of vector spaces and subspaces (3.2, 3.3)
- Definition of "span", "linear independence", "basis", and "dimension" (3.4, 3.5). You should know the exact definitions! This doesn't necessarily mean memorizing them word-for-word, and I won't ask you to repeat the definitions in the text. But each detail in these definitions is important. Thus, for example, a statement such as "A set $S = \{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_2\}$ of vectors is linearly independent if $a_1\mathbf{v}_1 + \cdots + a_n\mathbf{v}_n = \mathbf{0}$ and $a_1 = \cdots = a_n = 0$" is not only incorrect, it's meaningless.

As long as you're reading this, I might as well go ahead and give the correct definitions here. Suppose $S = \{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_2\}$ is a set of vectors in a vector space V. Suppose W is a subspace of V. That is, W is a set of vectors in V which is closed under addition and scalar multiplication.

The statement "S spans W" means that for every vector $\mathbf{v}$ in W, there exist real numbers $a_1, \ldots, a_n$ such that $a_1\mathbf{v}_1 + \cdots + a_n\mathbf{v}_n = \mathbf{v}$.

The statement "S is linearly independent" means that if $a_1, a_2, \ldots\ a_n$ are real numbers such that $a_1\mathbf{v}_1 + \cdots + a_n\mathbf{v}_n = \mathbf{0}$, then $a_1 = a_2 = \cdots = a_n = 0$.

The statement "S is a basis of W" means that S spans W and S is linearly independent.

Although a given subspace W has many different bases, it can be proved that all these bases must have the same number of elements. This makes it possible to unambiguously define the "dimension of W" to be the number of elements in a basis of W.

All of the above definitions apply to any subspace W of V. In particular, since any vector space is a subspace of itself, the definitions apply if W is replaced everywhere by V. That is, we define the statements "S spans V" and "S is a basis of V" in the same way as above, just replacing W by V throughout.

- Solution space of a system, null space of a matrix, nullity of a matrix (3.6, see also example 10 of 3.3).
- Coordinates of a vector with respect to a basis, transition matrices (3.7).

- Definition and basic properties of inner products (4.3, and Theorem 4.5 of section 4.4).
- Definition and basic properties of linear transformations. Standard matrix representation. (5.1)
- Kernel and range of a linear transformation (5.2)
- Matrix of a linear transformation $L : V \to W$ with respect to a basis S of the domain space V and a basis T of the range space W (5.3). In chapter 7, we usually deal with the case when the domain space and the range space are the same, and we represent L by a matrix using the same basis S for both the domain space and the range space. In this case we just say that the matrix represents L with respect to S.
- Determinants of 2×2 and 3×3 matrices (6.1). You should also be aware that a matrix A is nonsingular if and only if its determinant is zero. Other than this, you won't need to know anything else from chapter 6.
- Eigenvalues and eigenvectors, and how to find them (7.1).

I mentioned in class that I won't ask a problem on the final about the Gram-Schmidt procedure. I also won't ask anything about similarity of matrices (which was discussed in 5.5 and 7.2).