Review for Third Exam

The third exam covers sections 11.1, 11.2, 11.3, 11.4, 12.1, 12.2, 12.3, 12.4, and 12.5 of the textbook. The relevant assignments and quizzes are assignments 9 through 12, and quizzes 5 and 6.

11.1 To describe a curve in the xy-plane, instead of giving x as a function of y, or y as a function of x, it is often useful to give both x and y as functions of a third variable t, called a parameter. You can then plot points on the curve by picking various values of t and plotting the corresponding values of x and y (you do not use the value of t when plotting the point). Important examples of parametric equations for curves are the parametric equations for a circle and for a cycloid given in Examples 2 and 7 of this section.

11.2 Suppose you are given parametric equations for a curve — how would you compute its slope, or the area under the curve, or its arc length, or the surface area obtained by revolving the curve around an axis? Formulas were given for all these quantities earlier in the text, which you could use when the curve was described by giving y as a function of x. This section describes how to use the formulas when x and y are given as functions of a parameter. Note that for arc length and surface area, it is easy to get the correct formulas in parametric form from the ones given in x-y form, by using the substitution

$$ds = \sqrt{dx^2 + dy^2} = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt.$$

11.3 This section introduces polar coordinates, which are related to rectangular (x-y) coordinates by the equations in the red boxes labelled 1 and 2. You can remember these equations easily by remembering the diagram in Figure 5. You should familiarize yourself with the graphs of a few of the most basic polar equations, such as the ones in Examples 2, 6, and 9.

11.4 This section gives formulas for area and arc length in polar coordinates. I recommend memorizing them (equations in the red boxes labelled 4 and 5). You can also use geometry to help you remember these formulas: formula 4 comes from the formula for the area of a thin circular sector, and formula 5 comes from the Pythagorean theorem applied to a little right triangle on the curve (as explained in class).

12.1 Actually, you can skip this section if you like. It discusses what it means to take the limit of a sequence of numbers as n goes to infinity. But we've already covered limits of functions of x as x goes to infinity earlier in class, and this is essentially the same thing. For example, the limit of $\dfrac{n^2+1}{n^2+2}$ as n goes to infinity has the same value as the limit of $\dfrac{x^2+1}{x^2+2}$ as x goes to infinity.

12.2 This section contains the definition of what an infinite series is. You should understand this clearly; in particular you have to be aware of the distinction between, say, the limit of $1/n^2$ as n goes to infinity (which is zero), and the sum of $1/n^2$ from $n = 1$ to infinity (which is not zero). It's also important to study Examples 1 and 7 (the harmonic and geometric series) carefully, and to understand the Test for Divergence (page 728) clearly. In particular, remember that the converse of the Test for Divergence is not true: knowing that the limit of a_n is zero does NOT tell you that the series $\sum a_n$ converges. The harmonic series is a case in point.

12.3, 12.4, 12.5 Remember the integral test, the comparison test, the limit comparison test, and the alternating series test, and study the examples in these section. You do not need to worry about "remainder estimates" (pages 737, 744, and 748); I will not ask for remainder estimates on this exam. (They are certainly important in applications, though!)

If you haven't already, you might check out Jason's useful table of convergence tests, posted for this course on learn.ou.edu.