Review for Third Exam

The third exam will cover sections 4.1, 4.2, 5.1, 5.2, 5.3 and 6.1 of the text. The relevant assignments are assignments 17 through 24.

We've already had enough questions on definitions on the second quiz. But there might be one or two questions asking you to prove theorems from class, taken from the following list:

- Be able to prove that if $\lim\limits_{x\to c} f = L$ and $\lim\limits_{x\to c} g = M$, then $\lim\limits_{x\to c}(f+g) = L+M$ (part of 4.2.4(a)).
- Be able to prove that compositions of continuous functions are continuous (5.2.6, but notice that the proof I gave in class was phrased quite differently).
- Be able to prove the product rule for derivatives (6.1.3(c)).

Here's a guide to the sections in the text covered on the exam:

• Section 4.1: You should profit by going over the whole section. Notice that in class I emphasized using the sequential criterion to prove basic things about limits, rather than using the ϵ-δ definition. So I didn't cover examples such as Example 4.1.7, and I won't expect you to do such examples either. All the examples done in Example 4.1.7 can be done using the sequential criterion instead.

The sequential criterion is a powerful tool, but has to be used carefully. To prove that a function has a limit at c using the sequential criterion, you have to consider *every* sequence (x_n) which approaches c; it's not enough to look at a single sequence. On the other hand, to prove that a function does not have a limit at c using the sequential criterion, it is enough to consider a single sequence (see 4.1.9).

• Section 4.2: You should read the entire section.

One common mistake made in connection with this section has to do with limits that do not exist. You cannot use Theorem 4.2.4(b) to show that $\lim_{x\to 0} = \frac{1}{x}$ does not exist. The fact that division by zero is undefined is irrelevant to the question of whether this limit exists. Rather, to prove this limit does not exist, you can use an argument like that in 4.1.10(a)

• Section 5.1: Re-read the whole section. Notice the useful Sequential Criterion for Continuity (5.1.3, to be distinguished from the Sequential Criterion for Limits, 4.1.8).

• Section 5.2: This short, yet important section should be re-read completely.

• Section 5.3: Read from the beginning of the section through Theorem 5.3.5. You can skip the remaining material in this section.

Although I proved the Location of Roots theorem (5.3.5) in class, and gave an example of how to use it, I didn't assign homework on this theorem, which is kind of a shame, because there are many nice problems that can be solved using this theorem. Anyway, the only question I might ask about this theorem would be a simple example like the one I gave in class (where I used the theorem to show that there exists a real number whose cube is 2).

• Section 6.1: Read up to (but not including) the subsection titled "Inverse Functions". You can skip this subsection (i.e., you can skip all the material after the heading "Inverse Functions").

• Section 6.2: This section will not be covered on the exam.