Review for Third Exam

The third exam will cover sections 16.4 through 17.4 of the text, not including sections 16.6 and 16.9. Here are a few comments on the material in these sections. Everything in these sections is possible material for the exam, unless excluded in the comments below. Also, I will not ask for proofs of any of the theorems in these sections.

16.4. Double integrals in polar coordinates. This section should be studied together with section 16.8. The important formulas to memorize are in the red boxes on p. 1041, p. 1069, and p. 1071. The diagrams on those pages are useful in helping you remember these formulas: they show why an element of volume in polar coordinates is given by $r\ dr\ d\theta$ and an element of volume in spherical coordinates is given by $\rho^2 \sin\phi\ d\rho\ d\phi\ d\theta$.

16.5. Applications of double integrals. You can omit the material on probability and expected values from this section. The formulas for moments and moments of inertia in this section should be compared with similar formulas in sections 16.7, 17.2 (see exercises 35 and 36), and for that matter section 17.7, although section 17.7 won't be covered on this exam. The common idea behind all these formulas is that the moments are obtained by integrating the product of a size element (length, area, or volume) and a quantity which measures the distance of the element from an axis or plane.

16.7. Triple integrals. The trickiest problems in this section involve figuring out how to change the limits of integration appropriately when changing the order of integration. See Example 3 for a good illustration of such a problem.

16.8. Triple integrals in cylindrical and spherical coordinates. See the comments on section 16.4 above.

17.1. Vector fields. This section just introduces the concept of a vector field (a "field of arrows", one arrow associated with each point of space; or, equivalently, a variable vector whose components are functions of the space coordinates x, y, and z), and defines an important type of vector field known as a *gradient vector field*, also called a *conservative vector field*. The main reason gradient vector fields are important is that their line integrals are easy to compute, using the fundamental theorem for line integrals in section 17.3. Not every vector field is a gradient vector field. (The easiest way to tell whether a vector field is a gradient vector field is to check whether its curl is zero: if a vector field has zero curl in a simply connected domain, then it is a gradient vector field — see Theorem 6 on p. 1114 and Theorem 4 on p. 1128. Actually, although the book does not mention it, the statement in Theorem 4 on p. 1128 is valid in any simply connected domain in $\mathbf{R}^3$. Remember that a "simply connected domain" is one in which any closed curve can be shrunk to a point without leaving the domain.)

17.2. Line integrals. There are basically two kinds of line integrals defined in this section: line integrals with respect to arclength, of the form $\int_C f\ ds$, and line integrals with respect to x, y, or z, of the form $\int_C P\ dx + Q\ dy + R\ dz$. You should also be familiar with the notation $\int_C \mathbf{F} \cdot d\mathbf{r} = \int_C P\ dx + Q\ dy + R\ dz$, called the line integral of the vector field $F = P\mathbf{i} + Q\mathbf{j} + R\mathbf{k}$ over the curve C. Line integrals of both types are evaluated by first parametrizing the curve C and then rewriting the line integral as an integral with respect to the parameter t. Study the examples in this section carefully to see how to evaluate line integrals.

17.3. The fundamental theorem for line integrals. The line integrals of a vector field are said to be independent of path if the value of the line integral over a curve C depends only on the beginning and endpoints of C, and not on what path C takes to get from its beginning to its endpoint. The fundamental theorem (red box on p. 1110) says that if $\mathbf{F}$ is a gradient vector field then its line integrals are independent of path, and moreover are easily calculated once you compute the function whose gradient is $\mathbf{F}$. Note Theorem 4 on p. 1112, which says that the line integrals of a vector field are independent of path if and only if the vector field is a gradient vector field. Thus independence of path, being a gradient field, and having zero curl all go together, at least in simply connected domains. (As noted in class, in domains which are NOT simply connected one can find vector fields which have zero curl but which are not gradient vector fields and whose line integrals are not independent of path.)

17.4. Green's Theorem. Make sure you know the statement of Green's Theorem (red box on p. 1119) and can do problems like the ones assigned for this section. To remember the right-hand side of Green's Theorem, it may be a help to learn it in the form given in equation 12 of the next section 17.5.