Review for Second Exam

The second exam covers sections 8.1, 8.2, 8.3, 8.4, 8.8, 9.1, 9.2, and 9.3 of the textbook.

To begin with, you might consider memorizing the integration formulas on page 488 of the text. This isn't strictly necessary, because you could in theory re-derive many of these formulas for yourself if you needed them. But I think it does make it a lot easier to do more complicated integrals if you have these simple ones memorized.

You should also make sure you're comfortable with integration by substitution (as in, for example, Examples 3 and 4 on page 335, Example 8 on page 401, and Examples 10 and 11 on page 415. You should at this point be able to do examples like these without even breaking a sweat.

1. Memorize the integration by parts formula (box number 2 on page 489). When using this formula you have to make a decision as to what part of the integrand to use for u and what part to use for dv. Study the examples in Section 8.1 to try to get a feel for how to make this decision.

2. Don't bother trying to memorize all the different rules or "strategies" listed in Section 8.2. Instead, just remember that for trigonometric integrals, it's usually a good idea to make one of the substitutions $u = \sin x$, $u = \tan x$, or $u = \sec x$; and in some cases you might have to use one of the half-angle identities listed in part (c) of the red box on page 498. Knowing this, and having gone through a few examples, should be enough to allow you to do any trig integral that comes up.

3. Section 8.3 is actually one of the easiest sections in the book, because everything here is completely mechanical; no insight or creativity is required. Just use the appropriate substitution from the table at the bottom of page 503 to get the integral into the form of a trigonometric integral. Evaluate the trigonometric integral (well, that part might not always be so easy, but that's material from section 8.2, not 8.3), and then use a diagram like the ones in Figure 3 or Figure 4 of this section to rewrite the result in terms of the original variable of integation.

4. Section 8.4 is just about as mechanical, and consequently easy, as section 8.3. Read Examples 1 through 6. You do not need to read any of the remainder of the section; in particular, there will not be any integrals with repeated irreducible quadratic factors on the exam.

5. Arclength is given by $L = \int\ ds$, and the surface area of a solid of revolution is given by $S = \int 2\pi x\ ds$, if the surface is generated by revolving a curve around the y-axis. (A surface generated by revolving a curve around the x-axis would have area given by $S = \int 2\pi y\ ds$.) Here the "ds" stands for $\sqrt{dx^2 + dy^2}$, which in turn is supposed to be interpreted as either $\sqrt{1 + (dy/dx)^2}\ dx$ or $\sqrt{(dx/dy)^2 + 1}\ dy$, depending on whether you want to do the integral with respect to x or y. In solving problems using these formulas, it helps to understand where they come from: not only does that free you from having to rely completely on memory, it gives you ways to check your computations as you go along, by seeing if the values of the variables are consistent with their geometric meaning.

6. Formulas for the moments M_x and M_y of a plane solid, the mass m of the solid, and the coordinates $\bar{x}$ and $\bar{y}$ of the solid's center of mass are given in the three red boxes on pages 580, 581, and 582. Actually the equations for M_x, M_y, and m differ from the ones I gave in class, which were

$$
\begin{aligned}
M_x &= \int_c^d y\rho(x_1 - x_2)\ dy,\\
M_y &= \int_a^b x\rho(y_1 - y_2)\ dx,\\
m &= \int_c^d \rho(x_1 - x_2)\ dy = \int_a^b \rho(y_1 - y_2)\ dx.
\end{aligned}
$$

The above formulas don't assume that the density ρ of the solid is constant; that is, they can be used even if ρ is a function of x and/or y. In the case when ρ is constant, you can factor it out of the above integrals, and you get the same formula as in the text for M_y. The formula for M_x in the text looks quite different from the one given above, but they are both correct and (in case the density is constant) will both give the same result. You can use whichever formulas you like on the exam. I will not ask any questions about solids in which the density is not constant.