Review for First Exam (part 2)

Here is a summary of things you should know for the first exam from sections 7.6, 7.7, and 7.8 of the textbook.

1. The notation "$y = \arcsin x$" or "$y = \sin^{-1} x$" means that $x = \sin y$. In English: the arcsine of x is (the radian measure of) the angle whose sine is x. Thus, for example, the arcsine of 1 is $\pi/2$, because $\pi/2$ is the radian measure of a 90° angle, whose sine is 1. (The notation $y = \sin^{-1} x$ is, unfortunately, confusing. It does *not* mean that $y = 1/(\sin x)$.)

The arcsine function as defined above is multi-valued, because there are many different angles whose sine is a given number x. In this course, to make the arcsine function single-valued, we restrict its values to lie between $-\pi/2$ and $\pi/2$. Perhaps the easiest way to remember this is to remember what the graph of the "principal branch" of the arcsine function is: see Figure 4 on p. 455.

The arctangent and arcsecant functions are defined similarly to the arcsine function: $y = \tan^{-1} x$ means that $x = \tan y$, and $y = \sec^{-1} x$ means that $x = \sec y$. To make them into single-valued functions, we require that the value of the arctangent lie between $-\pi/2$ and $\pi/2$, and we require that the value of the arcsecant lie either between 0 and $\pi/2$ or between π and $3\pi/2$. You won't need to remember those restrictions for the test, however; if they become an issue on any of the test problems, I'll remind you of them on the test.

What the arcsine, arctangent, and arcsecant have in common with the logarithm (and with the inverse hyperbolic functions discussed below) is that they are all inverses of functions that you are already familiar with. When we speak of the inverse of a function, we're not referring to inverses in the sense that 1/2 is the inverse of 2. Rather, when we say that the function g is the inverse of the function f, we mean that "$y = g(x)$" is equivalent to "$x = f(y)$". So the natural logarithm function is the inverse of the exponential function to the base e, in the sense that $y = \ln x$ means the same as $x = e^y$.

2. You should memorize the formulas for the derivatives of the arcsine, arctangent, and arcsecant functions (see the red box on p. 459). You should also be able to derive these formulas on the test, if asked. The derivations are all the same, and are similar to the derivation of the formula for the derivative of $\ln x$: you start by setting y equal to the inverse function whose derivative you want to find; then find dy/dx using implicit differentiation. To express the answer in terms of x, you will need to use the trig identities

$$\sin^2 x + \cos^2 x = 1$$

and

$$\tan^2 x + 1 = \sec^2 x,$$

which I recommend that you memorize as well.

Once you have these derivative formulas memorized, then you will automatically know the corresponding integration formulas, two of which are listed as formulas 12 and 13 on p. 460. You should also study examples 7 and 8, which show how to use integration by substitution to find related integration formulas.

3. You should remember the definition of the hyperbolic sine, cosine, tangent, and secant functions ($\sinh x$, $\cosh x$, $\tanh x$, and $\text{sech}\, x$), which are given in the red box on p. 463. You should also memorize the identity

$$\cosh^2 x - 1 = \sinh^2 x,$$

and the derivatives of the hyperbolic sine and cosine functions:

$$\frac{d}{dx}(\sinh x) = \cosh x, \qquad \frac{d}{dx}(\cosh x) = \sinh x.$$

You do not need to memorize the derivatives of the other hyperbolic functions, or the derivatives of the inverse hyperbolic functions (red box on p. 467). If for some reason you ever wanted to figure out the derivatives of the inverse hyperbolic functions, you could do so by the same procedure used to find the derivatives of the inverse trigonometric functions (see above).

4. You already know how to use L'hopital's rule: this is one rule that everybody remembers from the very first moment they see it. What you have to be careful to remember, rather, is to limit your use of the rule to situations where it applies: it *only* applies to limits which are in the form $\frac{0}{0}$ or $\frac{\infty}{\infty}$.

Some limits which are in the form $0 \cdot \infty$, or 0^0, or 1^∞, or ∞^0, or $\infty - \infty$, can be evaluated with the help of L'hopital's rule, but as a first step you must always find a way to rewrite the limit (or the logarithm of the limit) in the form $\frac{0}{0}$ or $\frac{\infty}{\infty}$. See examples 6, 7, 8, 9, 10 in section 7.8; as well as a few similar examples I did in class.